P1

Se sabe que $V = \omega d$ y por geometría

$$\operatorname{Sen}\theta = \frac{d}{r} \Rightarrow d = \operatorname{Sen}\theta\, r$$

Reemplazando: $V = \omega \operatorname{Sen}\theta\, r$ $\quad |A \times B| = AB\operatorname{Sen}\theta$

$$\Rightarrow V = \omega \times r_{\perp}$$

$$\omega \times r = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & \omega \\ x & y & z \end{vmatrix} = \langle -y\omega, x\omega, 0 \rangle = V$$

$$\nabla \times V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ -y\omega & x\omega & 0 \end{vmatrix} = \langle 0, 0, \omega - (-\omega) \rangle = 2\omega\hat{k}$$

//

P2

$$\nabla r = \nabla\left(\sqrt{x^2+y^2+z^2}\right) = \left\langle \frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}} \right\rangle$$

$$\mathbf{R}/r = \frac{\langle x, y, z\rangle}{\sqrt{x^2+y^2+z^2}} \Rightarrow \boxed{\mathbf{R}/r = \nabla r}$$

$$\nabla \frac{1}{r} = \nabla\left(\frac{1}{\sqrt{x^2+y^2+z^2}}\right) = \left\langle \frac{\frac{2x}{-2\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}, \frac{\frac{2y}{-2\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2}, \frac{\frac{2z}{-2\sqrt{x^2+y^2+z^2}}}{x^2+y^2+z^2} \right\rangle$$

$$\Rightarrow \left\langle \frac{-x}{(x^2+y^2+z^2)^{3/2}}, \frac{-y}{(x^2+y^2+z^2)^{3/2}}, \frac{-z}{(x^2+y^2+z^2)^{3/2}} \right\rangle$$

$$-\mathbf{R}/r^3 = \langle -x, -y, -z\rangle \cdot \frac{1}{(x^2+y^2+z^2)^{3/2}} \Rightarrow \boxed{\nabla \frac{1}{r} = -\frac{\mathbf{R}}{r^3}}$$

$$\nabla \times r = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix} = \langle 0, 0, 0\rangle = \vec{0}_{//}$$

$$\nabla \ln r = \nabla \ln\left(\sqrt{x^2+y^2+z^2}\right) = \frac{1}{2}\nabla \ln\left(x^2+y^2+z^2\right) \Rightarrow$$

$$\frac{1}{2}\left\langle \frac{2x}{x^2+y^2+z^2}, \frac{2y}{x^2+y^2+z^2}, \frac{2z}{x^2+y^2+z^2} \right\rangle = \left\langle \frac{x}{x^2+y^2+z^2}, \frac{y}{x^2+y^2+z^2}, \frac{z}{x^2+y^2+z^2} \right\rangle$$

$$\mathbf{R}/r^2 = \langle x, y, z\rangle \cdot \frac{1}{x^2+y^2+z^2} \Rightarrow \boxed{\nabla \ln r = \frac{\mathbf{R}}{r^2}}$$

P3)

$$F = \langle P_1, Q_1, R_1 \rangle \wedge Q = \langle P_2, Q_2, R_2 \rangle$$

1) $$\frac{\partial P_1}{\partial x} + \frac{\partial P_2}{\partial x} + \frac{\partial Q_1}{\partial y} + \frac{\partial Q_2}{\partial y} + \frac{\partial R_1}{\partial z} + \frac{\partial R_2}{\partial z}$$

$$\Rightarrow \frac{\partial P_1}{\partial x} + \frac{\partial Q_1}{\partial y} + \frac{\partial R_1}{\partial z} + \frac{\partial P_2}{\partial x} + \frac{\partial Q_2}{\partial y} + \frac{\partial R_2}{\partial z}$$

$$\Rightarrow DIV(P) + DIV(Q)$$

2) Procedimiento igual a 1)

3) $$DIV\left(f_{(x,y,z)} F_{(x,y,z)}\right) = DIV\left(f F_{(x,y,z)}\right)$$

$$DIV\left(\langle f P_1, f Q_1, f R_1 \rangle\right)$$

$$\frac{\partial f}{\partial x} P_1 + \frac{\partial P_1}{\partial x} f + \frac{\partial f}{\partial y} Q_1 + \frac{\partial Q_1}{\partial y} f + \frac{\partial f}{\partial z} R_1 + \frac{\partial R_1}{\partial z} f$$

$$\frac{\partial f}{\partial x} P_1 + \frac{\partial P_1}{\partial x} f + \frac{\partial f}{\partial y} Q_1 + \frac{\partial Q_1}{\partial y} f + \frac{\partial f}{\partial z} R_1 + \frac{\partial R_1}{\partial z} f$$

$$f\left(\frac{\partial P_1}{\partial x} + \frac{\partial Q_1}{\partial y} + \frac{\partial R_1}{\partial z}\right) + \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle \cdot \langle P_1, Q_1, R_1 \rangle$$

$$\Rightarrow f \cdot \operatorname{DIV}(F) + \operatorname{DIV}(f) \cdot F$$

4) PROCEDIMIENTO IGUAL A 3)

5) $\operatorname{DIV}(F \times G) = \nabla \cdot (F \times G)$

COMO HAY QUE APLICAR ∇ A CADA COMPONENTE DE $F \times G$, ESCRIBIMOS MÁS COMPACTO $\nabla \cdot (F \times G)$

$$= \begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P_1 & Q_1 & R_1 \\ P_2 & Q_2 & R_2 \end{vmatrix}$$

→ ESTO ES ∇

↳ ESTO ES $F \times G$

$$\Rightarrow \frac{\partial}{\partial x}\begin{vmatrix} Q_1 & R_1 \\ Q_2 & R_2 \end{vmatrix} - \frac{\partial}{\partial y}\begin{vmatrix} P_1 & R_1 \\ P_2 & R_2 \end{vmatrix} + \frac{\partial}{\partial z}\begin{vmatrix} P_1 & Q_1 \\ P_2 & Q_2 \end{vmatrix}$$

$$\Rightarrow \left[Q_1 \frac{\partial R_2}{\partial x} + R_2 \frac{\partial Q_1}{\partial x} - Q_2 \frac{\partial R_1}{\partial x} - R_1 \frac{\partial Q_2}{\partial x}\right] - \left[P_1 \frac{\partial R_2}{\partial y} + R_2 \frac{\partial P_1}{\partial y} - P_2 \frac{\partial R_1}{\partial y} - R_1 \frac{\partial P_2}{\partial y}\right]$$

$$+ \left[P_1 \frac{\partial Q_2}{\partial z} + Q_2 \frac{\partial P_1}{\partial z} - P_2 \frac{\partial Q_1}{\partial z} - Q_1 \frac{\partial P_2}{\partial z}\right]$$

$$\Rightarrow \left[P_2\left(\frac{\partial R_1}{\partial y} - \frac{\partial Q_1}{\partial z}\right) + Q_2\left(\frac{\partial P_1}{\partial z} - \frac{\partial R_1}{\partial x}\right) + R_2\left(\frac{\partial Q_1}{\partial x} - \frac{\partial P_1}{\partial y}\right)\right]$$

$$- \left[P_1\left(\frac{\partial R_2}{\partial y} - \frac{\partial Q_2}{\partial z}\right) + Q_1\left(\frac{\partial P_2}{\partial z} - \frac{\partial R_2}{\partial x}\right) + R_1\left(\frac{\partial Q_2}{\partial x} - \frac{\partial P_2}{\partial y}\right)\right]$$

$$\Rightarrow G \cdot \text{ROT}\, F \quad - F \cdot \text{ROT}\, G$$

6) $\text{DIV}(\nabla f \times \nabla g) = 0$

USAMOS LA FORMA COMPACTA DE ANTES

$$\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix}$$

$$\Rightarrow \frac{\partial}{\partial x}\begin{vmatrix} f_y & f_z \\ g_y & g_z \end{vmatrix} - \frac{\partial}{\partial y}\begin{vmatrix} f_x & f_z \\ g_x & g_z \end{vmatrix} + \frac{\partial}{\partial z}\begin{vmatrix} f_x & f_y \\ g_x & g_y \end{vmatrix}$$

$$\Rightarrow \frac{\partial}{\partial x}\left[f_y g_z - f_z g_y\right] - \frac{\partial}{\partial y}\left[f_x g_z - f_z g_x\right] + \frac{\partial}{\partial z}\left[f_x g_y - f_y g_x\right]$$

$$\Rightarrow f_{yx} g_z - f_y g_{zx} - f_{zx} g_y - f_z g_{yx} - \left(f_{xy} g_z + f_x g_{zy} - f_{zy} g_x - f_z g_{xy}\right)$$

$$+ f_{xz} g_y + f_x g_{yz} - g_{xz} f_y - g_x f_{yz}$$

$$f_{xy} g_z - f_y g_{xz} - f_{zx} g_y - f_z g_{xy} - f_{xy} g_z - f_x g_{zy} + g_{xy} f_z + f_{zy} g_x + g_{zx} f_y + f_x g_{yz} - g_{zx} f_y - g_x f_{yz}$$

USANDO TEOREMA CLAIRAUT. (DERIVADAS CRUZADAS IGUALES, $h_{xy} = h_{yx}$)

$$f_{xy} g_z - f_y g_{xz} - f_{zx} g_y - f_z g_{xy} - f_{xy} g_z - f_x g_{zy} + g_{xy} f_z + f_{zy} g_x + g_{zx} f_y + f_x g_{yz} - g_{zx} f_y - g_x f_{yz}$$

CADA COLOR SE VA A 0, YA QUE SON DE SIGNO CONTRARIO.

$\rightarrow f_{xy} g_z - f_{xy} g_z = 0$ 😊

¡¡ TODO DA CERO.

7) VEAMOS LA PARTE IZQUIERDA:

$$\nabla \times (\nabla \times F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} & \frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} & \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \end{vmatrix}$$

$$= \langle Q_{yx} - P_{yy} - P_{zz} + R_{zx},\ R_{zy} - Q_{zz} - Q_{xx} + P_{xy},\ P_{xz} - R_{xx} - R_{yy} + Q_{yz} \rangle$$

Ahora la derecha

$$\nabla(\nabla \cdot F) - \nabla^2 F = \langle P_{xx} + Q_{xy} + R_{xz},\ P_{yx} + Q_{yy} + R_{yz},\ P_{zx} + Q_{zy} + R_{zz} \rangle$$

$$- \langle P_{xx} + P_{yy} + P_{zz},\ Q_{xy} + Q_{yy} + Q_{zz},\ R_{zx} + R_{zy} + R_{zz} \rangle$$

$$= \langle Q_{xy} + R_{xz} - P_{yy} - P_{zz},\ P_{yx} + R_{yz} - Q_{xx} - Q_{zz},\ P_{zx} + Q_{zy} - R_{xx} - R_{yy} \rangle$$

Usando Clairaut, las derivadas cruzadas son iguales y ∴ se tiene la igualdad pedida

P4) PD: $\iint_D f \nabla^2 g \, dA = \oint_C f(\nabla g) \cdot n \, ds - \iint_D \nabla f \cdot \nabla g \, dA$

$$\oint_C f(\nabla g) \cdot n \, ds = \iint_D \operatorname{DIV}(f(\nabla g)) \, dA$$

$$\oint_C f(\nabla g) \cdot n \, ds = \iint_D (\nabla \cdot f)(\nabla g) + f \nabla^2 g \, dA$$

$$\oint_C f(\nabla g) \cdot n \, ds = \iint_D \nabla f \nabla g \, dA + \iint_D f \nabla^2 g \, dA$$

$$\oint_C f(\nabla g) \cdot n \, ds - \iint_D \nabla f \nabla g \, dA = \iint_D f \nabla^2 g \, dA$$

P5)

Recordar que para todo campo, $DIV(ROT) = 0$

Si existiera G, cumpliría con lo anterior

$$DIV(ROT\ G) = \frac{\partial}{\partial x} x\,Sen y + \frac{\partial}{\partial y} Cos y + \frac{\partial}{\partial z}(z - xy) \overset{?}{=} 0$$

$$\Rightarrow Sen y - Sen y + 1 \overset{?}{=} 0$$

$$\Rightarrow 1 \neq 0$$

Contradice $DIV(ROT) = 0$

Y ∴ No existe.

b) Calculamos el rot (F)

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f(x) & g(y) & h(z) \end{vmatrix} = \left\langle \frac{\partial}{\partial y}(h(z)) - \frac{\partial}{\partial z}(g(y)),\ \frac{\partial}{\partial z}(f(x)) - \frac{\partial}{\partial x}(h(z)),\ \frac{\partial}{\partial x}(g(y)) - \frac{\partial}{\partial y}(f(x)) \right\rangle$$

$$= \langle 0, 0, 0 \rangle$$

YA QUE SE DERIVAN TODO RESPECTO A LA OTRA VARIABLE.

P6

ELIMINADA

FALTA INFORMACIÓN