Problema 1

Sea F un campo conservativo en $\mathbb{R}^2$, cuyas componentes tiene derivadas parciales continuas, tal que $2\ r \cdot F + |r|^2\ divF = 1$ donde $r = x\hat{i} + y\hat{j}$ es el vector posición. Si C es una curva cuya longitud es 8, cerrada, simple, con vector unitario normal exterior n, que encierra una región D contenida en $\mathbb{R}^2$ cuya área es 4, entonces

$$\oint_C |r|^2\ F \cdot n ds$$

es igual a:

$$|r|^2 \vec{F} = r \cdot r\vec{F}$$

Se cumplen Hip. Green Vect.

$$\Rightarrow \oint \underbrace{|r|^2 \vec{F}}_{\vec{G}} ds = \iint_D DIV(\vec{G})\, dA$$

$$DIV(r \cdot rF) =$$

$$DIV(r) \cdot rF + r \cdot DIV(rF)$$

$$2 \cdot rF + r \cdot (DIV(r)F + rDIV(F))$$

$$+ t \cdot (2F + t\,\mathrm{DIV}(F))$$

$$+ t \cdot 2F + t \cdot t\,\mathrm{DIV}(F)$$

$$\underbrace{2t \cdot F + |t|^2\,\mathrm{DIV}(F)}_{1}$$

$$\mathrm{DIV}(G) = 1$$

$$\oint_C \langle t, F \rangle\, dt = \iint_D 1\, dA = 4 \;//$$

Por envolvente

Problema 2

Calcule

$$\int_\gamma \overbrace{(2\ x\ e^{x+y} + x^2\ e^{x+y} + y)}^{P}\ dx\ +\ \overbrace{x^2\ e^{x+y} + x}^{Q}\ dy$$

donde γ es el segmento de cicloide $\gamma(t) = (t - sen(t), 1 - cos(t))$ que va desde el punto $(2\pi, 0)$ al punto $(0, 0)$

Veamos si es conservativo

$$\frac{\partial P}{\partial y} = 2x e^{x+y} + x^2 e^{x+y} + 1$$

$$\frac{\partial Q}{\partial x} = 2x e^{x+y} + x^2 e^{x+y} + 1$$

Son iguales $\Rightarrow$ Cons. en todo $\mathbb{R}$.

$\alpha(\tau) = (2\pi, 0) + \big((0,0) - (2\pi, 0)\big)\,\tau \quad , \tau \in [0,1]$

$\alpha(\tau) = (2\pi - 2\pi\tau,\ 0) = (2\pi(1-\tau), 0)$

$$\int_\gamma F \cdot d\vec{r} = \int_0^1 4\pi(1-\tau) e^{2\pi(1-\tau)} + \cdots \quad \text{PASA!}$$

Encontremos f

Por inspección

$$f(x,y) = x^2 e^{x+y} + xy$$

$\Rightarrow$ por TFIL

$$f(H(b)) - f(H(a))$$

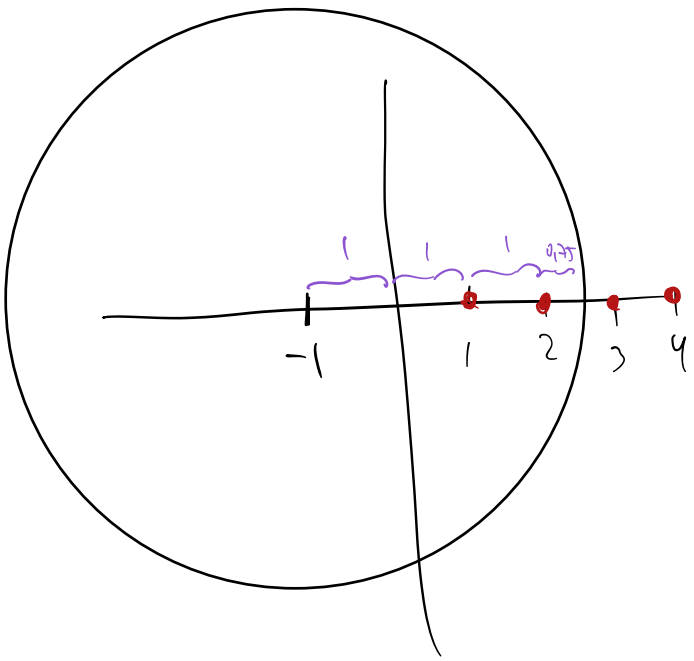
$$\Rightarrow f(H(1)) - f(H(0))$$

$$\Rightarrow f((0,0)) - f((2\pi,0))$$

$$\Rightarrow 0 - (2\pi)^2 e^{2\pi}$$

$$\Rightarrow -4\pi^2 e^{2\pi} \;//$$

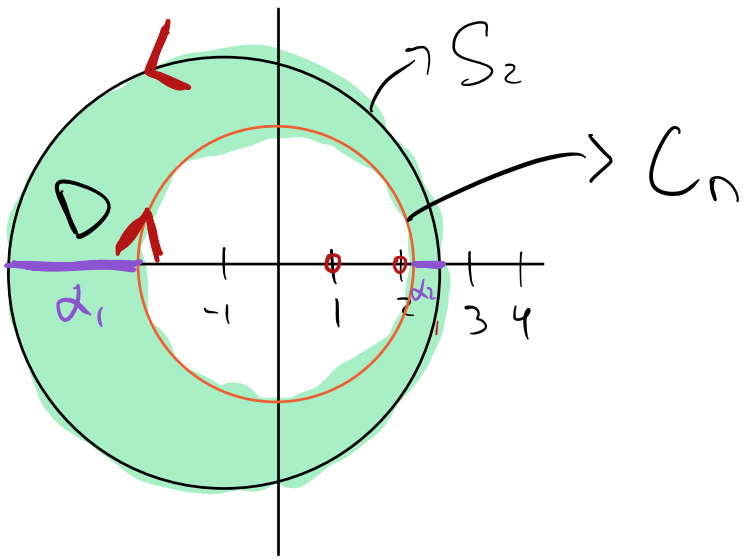
P3)



No sé wai es el campo, no quiero usar definición.

Green

↓



$$\oint_{S_2 \cup C_n^-} \vec{F} \cdot d\vec{r} = \iint_D \underbrace{\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}}_{\text{0 YA Que es cons.}} dA$$

$$\oint_{S_2} \vec{F} \, d\vec{r} + \oint_{C_n^-} \vec{F} \cdot d\vec{r} = 0$$

$$\oint_{S_2} \vec{F} \cdot d\vec{r} - \oint_{C_n} \vec{F} \cdot d\vec{r} = 0$$

$$\oint_{S_2} \vec{F} \cdot d\vec{r} = \oint_{C_n} \vec{F} \cdot d\vec{r}$$

¿Qué C_n contiene las 2 singularidades?

C_2, ya que $2+\frac{1}{2} = \boxed{2,5}$

$$\therefore \oint_{S_2} \vec{F}\cdot d\vec{r} = \oint_{C_2} \vec{F}\cdot d\vec{r}$$

Por enunciado, $\oint_{C_n} \vec{F}\,d\vec{r} = n$

$$\Rightarrow \oint_{C_2} \vec{F}\,d\vec{r} = 2$$

$$\therefore \oint_{S_2} \vec{F}\cdot d\vec{r} = 2\,\text{//}$$

Importante decir que las

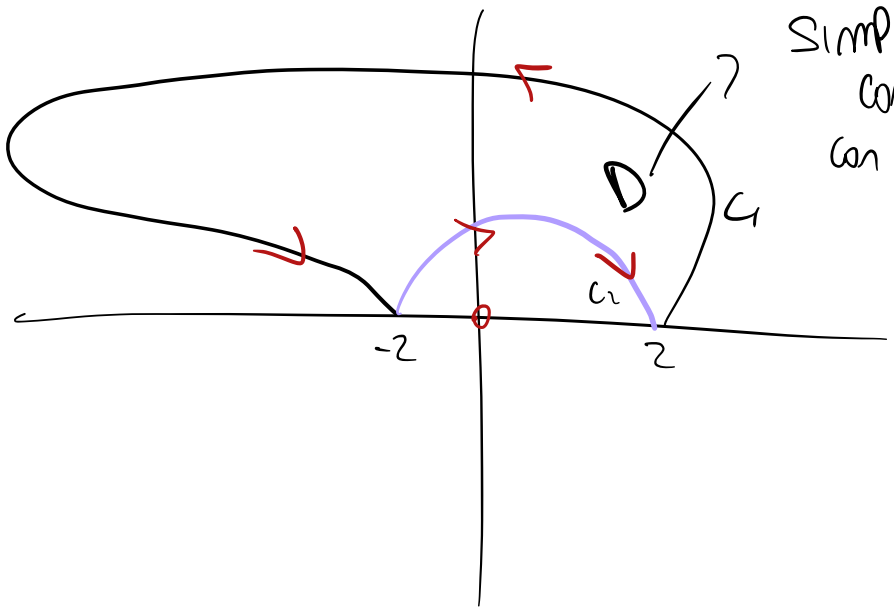
Curvas α_1 y α_2

en la intersección de líneas

se cambia mutuamente

P4)

Como hay singularidad, la evitamos



Simplemente conexa con $\frac{\partial}{\partial}$ continuas

$$\frac{\partial P}{\partial y} = \frac{2y^2}{(x^2+y^2)^2} - \frac{1}{x^2+y^2} = \frac{-x^2+y^2}{(x^2+y^2)^2}$$

$$\frac{\partial Q}{\partial x} = \frac{-2x^2}{(x^2+y^2)^2} + \frac{1}{x^2+y^2} = \frac{-x^2+y^2}{(x^2+y^2)^2}$$

$\therefore$ Son cyuales $\Rightarrow$ Conservativo en D.

$$\oint_{C_1 \cup C_2} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \, dA$$

$$\Rightarrow \oint_{C_1} + \oint_{C_2} = 0$$

$$\Rightarrow \oint_{C_1} = -\oint_{C_2}$$

$$\Rightarrow \oint_{C_1} = \oint_{C_2^-}$$

Parametrizando C_2

$$r(t) = \langle 2\cos t, 2\operatorname{sen} t \rangle, \quad t \in [0, \pi]$$

$$\Rightarrow \oint_{C_2^-} = \int_0^{\pi} \left\langle 3 - \frac{2\operatorname{sen}}{4\cos^2 + 9\operatorname{sen}^2}, 2 + \frac{2\cos t}{9\cos^2 + 9\operatorname{sen}^2} \right\rangle \cdot \langle -2\operatorname{sen}, 2\cos \rangle$$

$$\Rightarrow \int_0^{\pi} -6\operatorname{sen} + \operatorname{sen}^2 + 4\cos + \cos^2 \, dt$$

$$= \pi - 12$$

$$\therefore \quad \int_{C_1} F \cdot dr = \overline{\pi + 2}$$

P5

$$\frac{\partial f}{\partial x} = axy - z^3$$

$$\frac{\partial F}{\partial y} = (a-2)x^2$$

$$\frac{\partial f}{\partial z} = (1-a)xz^2$$

1)

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ axy - z^3 & (a-2)x^2 & (1-a)xz^2 \end{vmatrix}$$

$$= \left(0 - 0, \underbrace{-3z^2 - (1-a)z^2}, (a-2)2x - ax \right)$$

$$-3 = 1-a$$

$$\boxed{a = 4}$$

$$2ax - 4x - ax = 0$$

$$ax = 4x \Rightarrow a = 4$$

$\therefore \quad a = 4 \iff \nabla f$.

$$f(x,y,z) = \frac{ax^2y}{2} - z^3x + h(y,z)$$

$$\frac{\partial f}{\partial y} = \frac{ax^2}{2} + \frac{\partial h}{\partial y}$$

$$(a-2)x^2 = \frac{ax^2}{2} + \frac{\partial h}{\partial y}$$

$$a - 2 = \frac{a}{2}$$

$$2a - 4 = a$$

$$\boxed{a = 4}$$

$$\frac{\partial h}{\partial y} = 0$$

$$h(z)$$

$$f(x,y,z) = \frac{a}{2}x^2y - z^3x + h(z)$$

$$\downarrow$$

$$\frac{\partial F}{\partial z} = -3z^2x + h'(z)$$

$$(1-a)xz^2 = -3z^2x + h'(z)$$

$$1-a = -3$$

$$\boxed{a = 4}$$

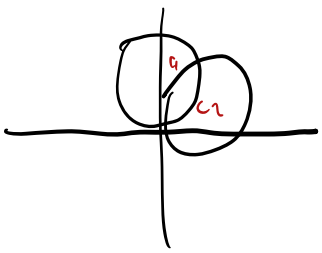
$$h'(z) = 0$$

$$\Rightarrow h(z) = C.$$

$$\therefore f(x,y,z) = 2x^2y - z^3x$$

P6)

NOTAR QUE SON CIRCUNFERENCIAS



$r = \sin\theta \Rightarrow r^2 = r\sin\theta$

$\Rightarrow x^2 + y^2 = y$

$\Rightarrow x^2 + (y - \frac{1}{2})^2 = \frac{1}{2}$

$r = \cos\theta \Rightarrow r^2 = r\cos\theta$

$\Rightarrow x^2 + y^2 = x$

$\Rightarrow (x - \frac{1}{2})^2 + y^2 = \frac{1}{2}$

LA CURVA ES LA INTER. DE C_1 Y C_2

$\Rightarrow$ PARA LAS HIP. DE GREEN

$$\oint_{C_1 \cup C_2} \vec{F} \cdot d\vec{r} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\, dA$$

$$\frac{\partial Q}{\partial x} = 2 \quad \wedge \quad \frac{\partial P}{\partial y} = -3$$

$\Rightarrow 2 - (-3) = 5$

$$\Rightarrow \int_{C_1 \cup C_2} \vec{F}\, d\vec{r} = \iint_D 5\, dA$$

$$= 5 \iint_D dA$$

$$= 5\left(\frac{1}{2}\int_0^{\pi/4} r_1^2(\theta)\, d\theta + \frac{1}{2}\int_{\pi/4}^{\pi/2} r_2^2(\theta)\, d\theta \right)$$

$$= \frac{5}{2}\left(\int_0^{\pi/4} \sin^2\theta\, d\theta + \int_{\pi/4}^{\pi/2} \cos^2\theta\, d\theta \right)$$

$$= \frac{5}{8}(\pi - 2) \quad //$$