P11

$b^2x^2 + y^2 = b^2 \rightarrow x^2 + \frac{y^2}{b^2} = 1$

$\rightarrow r(t) = \langle \cos t, b\sin t \rangle$

Donde $(-1,0)$ a $(1,0) \Rightarrow t \in [\pi, 0]$

$$\int_C \vec{F}(r(t)) \cdot r'(t)\,dt = \int_\pi^0 \langle 3b^2\sin^2 t + 2,\ 16\cos t \rangle \cdot \langle -\sin t, b\cos t \rangle\,dt$$

$$= \int_\pi^0 -3b^2\sin^3 t - 2\sin t + 16b\cos^2 t\,dt$$

$$= \left[(3b^2+2)\cos t - b^2\cos^3 t \right]\Big|_\pi^0 = 4b^2 - 8\pi b + 4$$

$\Rightarrow h(b) = 4b^2 - 8\pi b + 4 \quad / \frac{d}{db} \Rightarrow h'(b) = 0 = 8(b-\pi)$

$\therefore$ min en $b = \pi$

Y $W_{min} = 4(1-\pi^2)$

P2) $P(x,y) = ye^{xy}$ $\quad Q(x,y) = xe^{xy}$

$\frac{\partial P}{\partial y} = e^{xy} + xye^{xy}$ $\quad \frac{\partial Q}{\partial x} = e^{xy} + xye^{xy}$

$\therefore \vec{F}$ es conservativo $\Rightarrow F(x,y) = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$

Buscamos f tq $\nabla f = F$

Integrando P: $f(x,y) = e^{xy} + g(y)$

Derivando $f(x,y)$ respecto a y: $\frac{\partial f}{\partial y} = xe^{y} + g'(y)$

Igualando con $\frac{\partial Q}{\partial y}$: $xe^{y} + g'(y) = xe^{y}$

$\Rightarrow g'(y) = 0 \Rightarrow g(y) = C$

$f(x,y) = e^{xy} + C$

$\therefore f(x,y) = e^{xy}$ //

Como es conservativo, $\int_a^b \vec{F}(r(t)) \cdot r'(t)\,dt = f(r(b)) - f(r(a))$ (TFIL)

$r(0) = \left(1, \frac{\sqrt{2}}{2}\right) \quad \wedge \quad r(\pi/4) = (\sqrt{2}, 1)$

$$\Rightarrow \int_0^{\pi/4} \vec{F}(r(T)) \cdot r'(T)\, dT = f(r(\pi/4)) - f(r(0)) = e^{\sqrt{2}} - e^{\frac{\sqrt{2}}{2}}$$

P3

$$\int_C r'' \cdot dr = \int_a^b (x'', y'', z'') \cdot (x', y', z')\, dT$$

$$= \int_a^b x''x' + y''y' + z''z'\, dT \qquad \frac{d}{dT}x'^2 = 2x''x'$$

$$= \frac{1}{2}\int_a^b \frac{dx'^2}{dT} + \frac{dy'^2}{dT} + \frac{dz'^2}{dT}\, dT$$

Reconoan que $r(T) = \langle x(T), y(T), z(T)\rangle \Rightarrow r'(T) = \langle x'(T), y'(T), z'(T)\rangle$

$$\left(\frac{d\,|r'(T)|}{dT}\right)^2 = \frac{dx'^2}{dT} + \frac{dy'^2}{dT} + \frac{dz'^2}{dT}$$

$$= \frac{1}{2}\int_a^b \left(\frac{d\,|r'(T)|}{dT}\right)^2 dT$$

Como $r(T)$ está Arcoparam, $r'(T) = T'$

y $|T'| = 1 \Rightarrow \frac{d1}{dT} = 0$

$$\therefore \frac{1}{2}\int_a^b 0^2\, dT = 0 \;//$$

$$\int (r \times r') \cdot dr = \int_a^b \underbrace{((x,y,z) \times (x',y',z'))}_{\text{ENTREGA UN VECTOR}} \cdot r' \, dT$$

$\perp$ A (x,y,z) Y (x',y',z')

$$= \int_a^b \underbrace{(\text{Vector} \perp r \text{ y } r') \cdot r'}_{\text{EL PRODUCTO PUNTO ENTRE VECTORES } \perp \text{ ES 0.}} \, dT$$

$$\int_a^b 0 \, dT = 0 \; //$$

P4

$$P(x,y) = x^3 - \frac{x}{x^2+y^2} \rightarrow \frac{\partial P}{\partial y} = \frac{2xy}{(x^2+y^2)^2}$$

$$Q(x,y) = y^4 - \frac{y}{x^2+y^2} \rightarrow \frac{\partial Q}{\partial x} = \frac{2xy}{(x^2+y^2)^2}$$

CAMPO CONSERVATIVO $\mathbb{R}^2 - \{0,0\}$

Integrando P : $f(x,y) = -\frac{1}{2}\ln(x^2+y^2) + \frac{1}{4}x^4 + g(y)$

$$\frac{\partial f}{\partial y} = \frac{y}{x^2+y^2} + g'(y)$$

Igualando a Q:

$$\frac{y}{x^2+y^2} + g'(y) = \frac{y}{x^2+y^2} + y^4$$

$$\Rightarrow g'(y) = y^4$$

$$\Rightarrow g(y) = \frac{1}{5}y^5 + K$$

$$\Rightarrow f(x,y) = -\frac{1}{2}\ln(x^2+y^2) + \frac{1}{4}x^4 + \frac{1}{5}y^5 + K$$

$$\therefore f(x,y) = -\frac{1}{2}\ln(x^2+y^2) + \frac{1}{4}x^4 + \frac{1}{5}y^5$$

Parametrizamos un segmento C donde $(-1,1)$ a $(2,2)$

$$r(t) = (-1,1) + (\langle 2,2\rangle - \langle -1,1\rangle)t = (-1,1) + (3,1)t$$

$t \in [0,1]$

$$\Rightarrow \int_C \vec{F}\cdot d\vec{r} = f(r(1)) - f(r(0)) = -\frac{1}{2}\ln(2^2+2^2) + \frac{1}{4}2^4 + \frac{1}{5}2^5 - \left(-\frac{1}{2}\ln((-1)^2+1^2) + \frac{1}{4}(-1)^4 + \frac{1}{5}1^5\right)$$

$$= -\frac{1}{2}\ln(2^3) + 4 + \frac{32}{5} + \frac{1}{2}\ln(2) - \frac{1}{4} - \frac{1}{5}$$

$$= 4 + \frac{32}{5} - \frac{9}{20} - \ln(2)$$ //

P5 El campo es $F = P\,\hat{k}$ → Fuerza Peso

El paso o altura de la hélice se calcula con regla de 3.

$$\frac{6\pi}{30\,m} = \frac{T}{z\,m} \Rightarrow z = \frac{5}{\pi}T \quad , \quad T \in [0, 6\pi]$$

(3 rev)

Parametrizando: $r(T) = \langle 6\cos T,\ 6\sin T,\ \frac{5}{\pi}T \rangle$, $r'(T) = \langle -6\sin T,\ 6\cos T,\ \frac{5}{\pi} \rangle$

$$\int_C \vec{F}\cdot\vec{dr} = \int_0^{6\pi} \langle 0,\ 0,\ -(70+10) \rangle \cdot \langle -6\sin T,\ 6\cos T,\ \tfrac{5}{\pi} \rangle$$

(70 Kg hombre + pintura)

$$= \int_0^{6\pi} -80\cdot\frac{5}{\pi}\,dT = -\frac{400}{\pi}\cdot 6\pi$$

$$\therefore \int_C \vec{F}\cdot\vec{dr} = W = -2400\,J$$