**P1)** Si $F$ es conservativo $\Leftrightarrow \nabla f = F$

$\rightarrow \frac{\partial f}{\partial x} = ayz \quad ; \quad \frac{\partial f}{\partial y} = -2bxz \quad ; \quad \frac{\partial f}{\partial z} = 3c(z^2 + xy)$

Integrando $\frac{\partial f}{\partial x} \Rightarrow f(x,y,z) = axyz + h(y,z)$

Derivando $f(x,y,z) \Rightarrow \frac{\partial f}{\partial y} = axz + \frac{\partial h}{\partial y}$

$\Rightarrow axz + \frac{\partial h}{\partial y} = -2bxz$

Como en todo ejercicio de este tipo, lo marcado se debe eliminar $\Rightarrow a = -2b$

$\Rightarrow \frac{\partial h}{\partial y} = 0$

Integrando: $h(y,z) = g(z)$

$\Rightarrow f(x,y,z) = axyz + g(z)$

Derivando

$\Rightarrow$

$\Rightarrow \frac{\partial f}{\partial z} = axy + g' = 3C(z^2 + xy)$

$\Rightarrow axy + g' = 3Cz^2 + 3Cxy$

Lo mismo de Antes

$\Rightarrow a = 3C \quad \wedge \quad g' = 3Cz^2$

Integrando $g'$

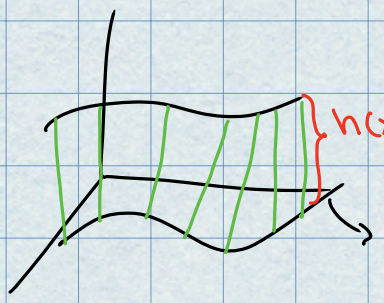
$\Rightarrow g(z) = 3z^3C$

$\Rightarrow f(x,y,z) = axyz + 3z^3C$

Con $a = 2$

$f(x,y,z) = 2xyz + \frac{2}{3}z^3$

P2)




$$\frac{100\ m^2}{1L} = \frac{A}{L} \Rightarrow L_{\text{litros}} = A \cdot \frac{1}{100}\ [L] \cdot 2$$

↑ Son 2 caras de la cerca.

La int. de línea de campo escalar es el área de la cortina vende.

$$\int_C \vec{F}\, d\vec{r} = \int_0^1 F(r(t))\, |r'(t)|\, dt$$

$$\Rightarrow \int_0^1 100t^2 \cdot 2t \sqrt{4t^2+4}\ dt$$

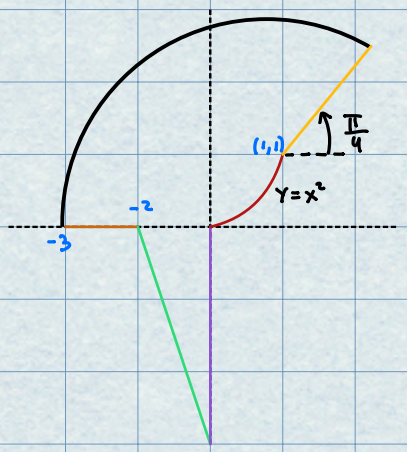
$$\Rightarrow 200 \int_0^1 t^3 \sqrt{4t^2+4}\ dt$$

$$\Rightarrow L = 4 \int_0^1 t^3 \sqrt{4t^2+1}\ dt$$

↳ reemplacé en $L = 2 \cdot \frac{A}{100}$

P3)

Parametrizamos cada parte de la figura.




$f = x$

$\rightarrow \gamma(t) = \langle t, t^2 \rangle \quad \wedge \quad \gamma' = \langle 1, 2t \rangle \quad \wedge \quad |\gamma'| = \sqrt{1+4t^2}$

$t \in [0,1]$

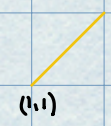
$$\int_0^1 t\sqrt{1+4t^2} = \frac{1}{12}(-1+5\sqrt{5})$$

$\rightarrow \gamma(t) = \langle 3\cos t, 3\operatorname{Sen} t \rangle \quad \wedge \quad |\gamma'| = 3$

$t \in [\pi/4, \pi]$

$$\int_{\pi/4}^{\pi} 3\cos t \cdot 3\, dt = -\frac{9}{\sqrt{2}}$$

$\left(\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right) = \frac{3\sqrt{2}}{2}(1,1)$



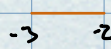

$\rightarrow \quad r(\tau) = (1,1) + \left[\frac{3\sqrt{2}}{2}(1,1) - (1,1)\right]\tau \quad , \quad \tau \in [0,1]$

$r(\tau) = (1,1) + \left(\frac{3}{\sqrt{2}} - 1, \frac{3}{\sqrt{2}} - 1\right)\tau$

$r(\tau) = \left(1 + \tau\left(\frac{3}{\sqrt{2}} - 1\right), \; 1 + \tau\left(\frac{3}{\sqrt{2}} - 1\right)\right)$

$|r'| = 3 - \sqrt{2}$

$$\int_0^1 \left(1 + \tau\left(\frac{3}{\sqrt{2}} - 1\right)\right)(3 - \sqrt{2})\, d\tau = \frac{7}{2\sqrt{2}}$$

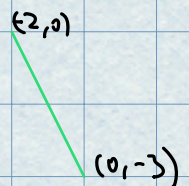



$\rightarrow \quad r(\tau) = (-3,0) + \left((-2,0) - (-3,0)\right)\tau$

$r(\tau) = (-3 + \tau, 0)$

$|r'| = 1$

$$\int_0^1 (-3 + \tau)\, 1 \, d\tau = -\frac{5}{2}$$

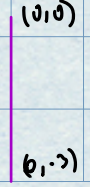



$r(\tau) = (-2,0) + \left((0,-3) - (-2,0)\right)\tau$

$r(\tau) = (-2 + 2\tau, -3\tau)$

$|r'| = \sqrt{13}$

$$\int_0^1 (-2 + 2\tau)\sqrt{13}\, d\tau = -\sqrt{13}$$




$r(\tau) = (0,-3) + (0,3)\tau$

$r(\tau) = (0, -3 + 3\tau)$

$|r'| = 3$

$\int_0^1 0 \cdot 3 = 0$ //

$\therefore$ El TOTAL es

$$\frac{1}{12}(-1+5\sqrt{5}) - \frac{9}{\sqrt{2}} + \frac{7}{2\sqrt{2}} - \frac{5}{2} - \sqrt{13} + 0$$

**P4)**

$$M = \int_C \rho(x,y,z)\, dl$$

$$\bar{x} = \frac{1}{M}\int_C \rho(x,y,z)\, x\, dl$$

$$\bar{x} = \frac{1}{M}\int_C \rho(x,y,z)\, y\, dl$$

$$\bar{x} = \frac{1}{M}\int_C \rho(x,y,z)\, z\, dl$$

---

$$\rho = x^2 + y^2 + z^2$$

$$r(t) = \langle \cos t, \operatorname{sen} t, t \rangle$$

$$|r'| = \sqrt{1+1} = \sqrt{2}$$

$$M = \int_0^{2\pi} (\cos^2 t + \sin^2 t + t^2) \cdot \sqrt{2}\, dt$$

$$= \int_0^{2\pi} \sqrt{2}t^2 + \sqrt{2} = \frac{2}{3}\sqrt{2}\,\pi(3 + 4\pi^2)$$

$$\bar{X} = \frac{3}{2\sqrt{2}\pi(3+4\pi^2)} \int_0^{2\pi} (\sqrt{2}t^2 + \sqrt{2}) \cos t \, dt$$

$$= \frac{3}{2\sqrt{2}\pi(3+4\pi^2)} \cdot 4\sqrt{2}\pi = \frac{6}{3 + 4\pi^2}$$

$$\bar{Y} = \frac{3}{2\sqrt{2}\pi(3+4\pi^2)} \int_0^{2\pi} (\sqrt{2}t^2 + \sqrt{2}) \sin t \, dt$$

$$= \frac{3}{2\sqrt{2}\pi(3+4\pi^2)} \cdot -4\sqrt{2}\pi^2 = \frac{-6\pi}{3+4\pi^2}$$

$$\bar{Z} = \frac{3}{2\sqrt{2}\pi(3+4\pi^2)} \int_0^{2\pi} (\sqrt{2}t^2 + \sqrt{2})\, t \, dt$$

$$= \frac{3}{2\sqrt{2}\pi(3+4\pi^2)} \cdot 2\sqrt{2}\pi^2(1 + 2\pi^2) = \frac{3(\pi + 2\pi^3)}{3+4\pi^2} //$$