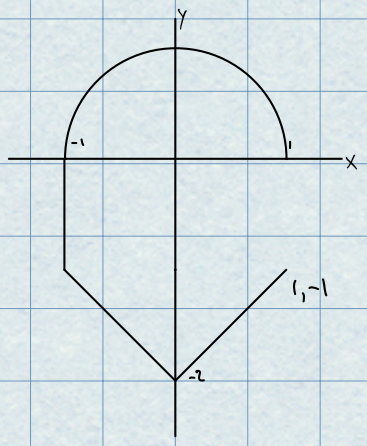
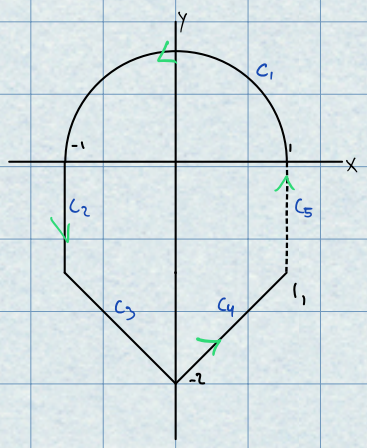
P1)



Curva γ

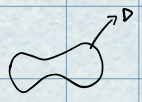
Queremos usar Green



① Cerrar la curva

② Enumerar partes

③ Verificar hipótesis Green

1- Región simplemente conexa 

2- Derivadas parciales continuas en D

3- Recorrida en sentido antihorario

$$\oint_{C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5} \vec{F} \cdot \vec{dr} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \, dA$$

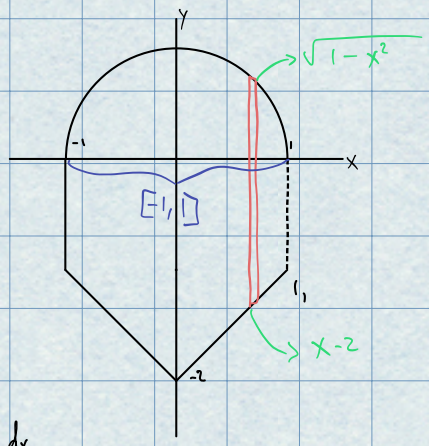
($C_1 \cup C_2 \cup C_3 \cup C_4 \cup C_5 = \gamma$)

$$\frac{\partial Q}{\partial x} = 6y^2 + 6x^2 \quad \wedge \quad \frac{\partial P}{\partial y} = 6y^2 - 2y$$

$$\Rightarrow \oint_{\gamma \cup C_5} \vec{F} \cdot \vec{dr} = \iint_D 6y^2 + 6xy - 6y^2 + 2y \, dA$$

$$= \iint_D 6xy^2 - 2y \, dA$$

$$= \int_{-1}^{1} \int_{x-2}^{\sqrt{1-x^2}} 6xy^2 - 2y \, dy \, dx$$



$$= \int_{-1}^{1} \underbrace{2x\left((1-x^2)^{3/2} - (x-2)^3\right)}_{\text{Función impar en un intervalo } -a,a \text{ integrado}} - \left((1-x^2) - (x-2)^2\right) dx$$

$$= -\int_{-1}^{1} 1 - x^2 - (x-2)^2 \, dx$$

Función par en intervalo $-a,a$ integrado $2 \cdot \int_0^a$

$= 2 \cdot A$

$$= -2 \int_0^1 -2x^2 + 4x - 3 \, dx$$

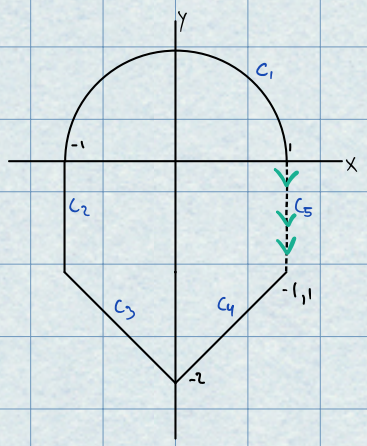
$$\oint_{\gamma \cup C_5} \vec{F} \cdot \vec{dr} = \frac{10}{3}$$

$$\Rightarrow \int_{\gamma} \vec{F} \cdot \vec{dr} + \int_{C_5} \vec{F} \cdot \vec{dr} = \frac{10}{3}$$

$$\Rightarrow \int_{\gamma} \vec{F} \cdot \vec{dr} = \frac{10}{3} - \int_{C_5} \vec{F} \cdot \vec{dr}$$

$$\int_{C} \vec{F} \cdot \vec{dr} = -\int_{-C} \vec{F} \cdot \vec{dr}$$

$$\Rightarrow \int_{\gamma} \vec{F} \cdot \vec{dr} = \frac{10}{3} + \int_{-C_5} \vec{F} \cdot \vec{dr}$$



PARA $-C_5$ DEBEMOS PARAMETRIZAR EN SENTIDO CONTRARIO

$$r_5(T) = (1,0) + (\langle 1,-1 \rangle - \langle 1,0 \rangle)T = (1,0) + (0,-1)T \quad , T \in [0,1]$$

$$r_5(T) = \langle 1, -T \rangle$$

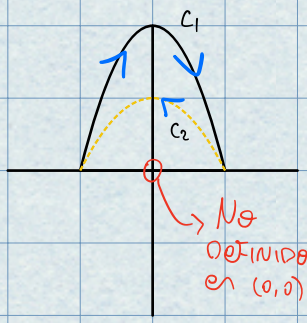
$$r_5'(T) = \langle 0, -1 \rangle$$

$$\int_{C_5} \vec{F} \cdot \vec{dr} = \int_0^1 \langle 2(-T-1)+T, 6T^2+3T^2 \rangle \cdot \langle 0,-1 \rangle \, dT = \int_0^1 -6T^2 - 3T^2 \, dT = \int_0^1 -9T^2$$

$$\int_{C_5} \vec{F} \cdot \vec{dr} = -3$$

$$\therefore \int_{\gamma} \vec{F} \cdot \vec{dr} = \frac{10}{3} - 3 = \frac{1}{3} \; //$$

P2) $r(\tau) = \langle \tau, 1-\tau^2 \rangle \Rightarrow x = \tau \wedge y = 1-x^2 \Rightarrow y(x) = 1-x^2$



Entregan la orientación así

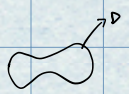
Queremos usar Green

① Cerrar la curva

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1- Región simplemente conexa



2- Derivadas parciales continuas en D

3- Recorrido en sentido antihorario

$$\oint_{-C_1 \cup -C_2} \vec{F} \cdot \vec{dr} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \, dA$$

$$\frac{\partial Q}{\partial x} = -\frac{\left((x^2+y^2) + (y-x)(2x)\right)}{(x^2+y^2)^2} \quad \wedge \quad \frac{\partial P}{\partial y} = \frac{1(x^2+y^2) - (x+y)(2y)}{(x^2+y^2)^2}$$

$$= \frac{x^2 - y^2 - 2xy}{(x^2+y^2)^2} \quad \wedge \quad = \frac{x^2 - y^2 - 2xy}{(x^2+y^2)^2}$$

entonces $\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 0$

$$\oint_{-C_1 \cup -C_2} \vec{F} \cdot \vec{dr} = \iint_D 0 \; dA = 0$$

$$\Rightarrow \int_{-C_1} \vec{F} \cdot \vec{dr} + \int_{-C_2} \vec{F} \cdot \vec{dr} = 0 \Rightarrow -\int_{C_1} \vec{F} \cdot \vec{dr} = \int_{C_2} \vec{F} \cdot \vec{dr}$$

Como C_2 es una semicircunferencia de radio 1 (o cualquiera) recorrida antihorario, $r_2(\tau) = \langle \cos\tau, \operatorname{sen}\tau \rangle$

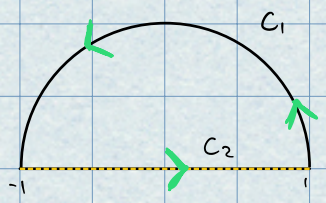
$$\int_{C_2} \vec{F} \cdot \vec{dr} = \int_0^{\pi} \left\langle \frac{\cos\tau + \operatorname{sen}\tau}{\cos^2\tau + \operatorname{sen}^2\tau}, \frac{\operatorname{sen}\tau - \cos\tau}{\cos^2\tau + \operatorname{sen}^2\tau} \right\rangle \cdot \langle -\operatorname{sen}\tau, \cos\tau \rangle \, d\tau$$

$$= \int_0^{\pi} -\operatorname{sen}\tau\cos\tau - \operatorname{sen}^2\tau + \cos\tau\operatorname{sen}\tau - \cos^2\tau \, d\tau$$

$$= -\pi$$

entonces, $-\int_{C_1} \vec{F} \cdot \vec{dr} = -\pi \Rightarrow \int_{C_1} \vec{F} \cdot \vec{dr} = \pi$ //

P3



$r_2(t) = (-1,0) + (\langle 1,0\rangle - \langle -1,0\rangle t) = (-1,0) + (2,0)t \quad , t \in [0,1]$

$r_2'(t) = (2,0)$

$\frac{\partial Q}{\partial x} = \frac{2+y}{1+y} \quad \wedge \quad \frac{\partial P}{\partial y} = \frac{1}{1+y}$

$\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = \frac{2+y}{1+y} - \frac{1}{1+y} = 1$

$\oint_{C_1 \cup C_2} \vec{F} \cdot \vec{dr} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \, dA$

$= \iint_D 1 \, dA$

Área zona D

$\int_{C_1} \vec{F} \cdot \vec{dr} + \int_{C_2} \vec{F} \cdot \vec{dr} = \frac{\pi}{2}$

$\int_{C_1} \vec{F} \cdot \vec{dr} = \frac{\pi}{2} - \int_{C_2} \vec{F} \cdot \vec{dr}$

$P_{ARA} \quad \int_{C_2} \vec{F} \cdot \vec{dr} = \int_0^1 \langle \ln(1), 2(2t-1) \rangle \cdot \langle 2,0 \rangle \, dt = \int_0^1 \langle 0, 2(2t-1) \rangle \cdot \langle 2,0 \rangle \, dt = 0$

$\therefore \quad \int_{C_1} \vec{F} \cdot \vec{dr} = \frac{\pi}{2}$

P4) Sea el campo $\vec{F}_{(x,y)} = \left\langle \frac{2xy}{(x^2+y^2)^2}, \frac{y^2-x^2}{(x^2+y^2)^2} \right\rangle$

Evalue $\oint_C \vec{F} \cdot \vec{dr}$ donde C es:

a) Circunferencia centrada en $(0,0)$ con radio $\frac{1}{2}$ y orientación antihorario

b) Hexágono regular centrado en $(0,0)$ con un vertice en $(1,0)$

Parametrizando a)

$$r(t) = \left\langle \tfrac{1}{2}\cos t, \tfrac{1}{2}\sin t \right\rangle, \quad t \in [0, 2\pi]$$

$$\oint_r \vec{F} \cdot \vec{dr} = \int_0^{2\pi} \left\langle \frac{2 \cdot \frac{1}{2} \cdot \frac{1}{2} \cos t \sin t}{\left(\left(\frac{1}{2}\cos t\right)^2 + \left(\frac{1}{2}\sin t\right)^2\right)^2}, \frac{\left(\frac{1}{2}\sin t\right)^2 - \left(\frac{1}{2}\cos t\right)^2}{\left(\left(\frac{1}{2}\cos t\right)^2 + \left(\frac{1}{2}\sin t\right)^2\right)^2} \right\rangle \cdot \left\langle -\tfrac{1}{2}\sin t, \tfrac{1}{2}\cos t \right\rangle$$

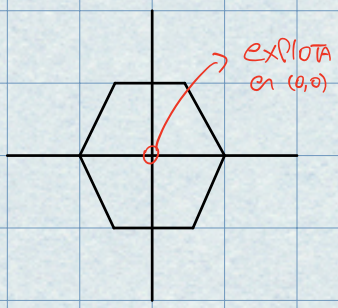
$$= \int_0^{2\pi} \left\langle 8\cos t \sin t, 8(\sin^2 t - \cos^2 t) \right\rangle \cdot \left\langle -\tfrac{1}{2}\sin t, \tfrac{1}{2}\cos t \right\rangle dt$$

$$= \int_0^{2\pi} -4\cos t \sin^2 t + 4\sin^2 t \cos t - 4\cos^3 t \, dt$$

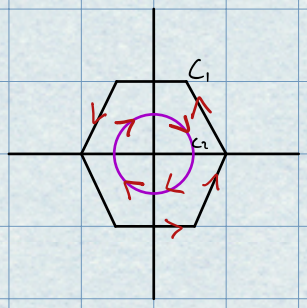
$$= \int_0^{2\pi} -4\cos^3 t = 0_{//}$$

$$\therefore \oint_r \vec{F} \cdot \vec{dr} = 0_{//}$$

AHORA, NOS PIDEN



NUESTRA ZONA POSEE UNA SINGULARIDAD, ENTONCES LA ENCERRAMOS CON LO QUE QUERAMOS



CUANDO HAY SINGULARIDADES, LAS ENCERRAMOS Y LAS ORIENTAMOS ANTIHORARIO, LUEGO USAMOS GREEN

$$\oint_{C_1 \cup C_2} \vec{F} \cdot \vec{dr} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \, dA$$

Notar que (a), $\oint_C \vec{F} \cdot d\vec{r} = 0$, lo que significa que el campo $\vec{F}$ es conservativo $\mathbb{R}^2 - \{0,0\}$

$\therefore$ $\vec{F}$ es conservativo $\Leftrightarrow \frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$

Entonces

$$\oint_{C_1 \cup C_2} \vec{F} \cdot d\vec{r} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \, dA$$

$$\oint_{C_1 \cup C_2} \vec{F} \cdot d\vec{r} = 0$$

$$\int_{C_1} \vec{F} \cdot d\vec{r} = -\int_{C_2} \vec{F} \cdot d\vec{r}$$

$\therefore$ $\int_{C_1} \vec{F} \cdot d\vec{r} = 0$ //