PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
FACULTAD DE MATEMATICAS
DEPARTAMENTO DE MATEMATICA
Segundo semestre 2023

Ayudantía 14- MAT1620

1. Use ordenadas polares para evaluar :

   a) $\iint_D e^{-x^2-y^2}\,dA$ donde D es la región acotada por la semicircunferencia $x = \sqrt{4-y^2}$ y el eje Y.

   b) $\int_0^1 \int_y^{\sqrt{2-y^2}} (x+y)\,dx\,dy$

2. Evalúe las siguientes integrales.

   (a) $\int_0^{\sqrt{\pi}} \int_0^x \int_0^{xz} x^2 \text{sen}(y)\,dy\,dz\,dx.$

   (b) $\iiint_E e^{z/y}\,dV$ donde $E = \{(x,y,z) : 1 \leq y \leq 4, y \leq z \leq 4, 0 \leq x \leq z\}$

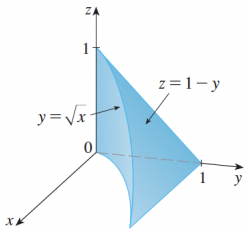
   (c) $\iiint_E x\,dV$, donde E es el sólido que está acotado por el paraboloide $x = 4y^2 + 4z^2$ y el plano $x = 4$.

3. Use integrales triples para calcular el volumen del sólido encerrado por el cilindro $x^2 + y^2 = 4$ y los planos $y = -1$ y $z + y = 4$.

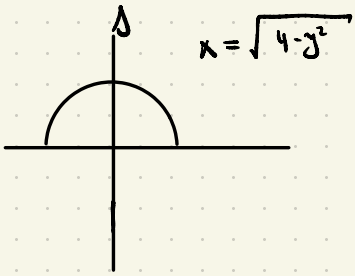
4. La figura muestra la región de integración para la integral

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)\,dz\,dy\,dx$$

   Reescriba en los otros cinco órdenes esta integral como una integral iterada equivalente.



a)



$$x=\sqrt{4-y^2}$$

$$x^2=4-y^2$$

$$x^2+y^2=4$$

$$x=r\cos\theta$$

$$y=r\operatorname{sen}\theta$$

$$0\le r\le 2$$

$$0\le\theta\le\pi$$

$$\int_0^{\pi}\int_0^{2} e^{-r^2}\, r\,dr\,d\theta = \int_0^{\pi} -\frac{e^{-r^2}}{2}\Big|_0^2 d\theta = \int_0^{\pi} -\frac{1}{2}\left(e^{-4}-e^{-0}\right)d\theta$$

$$\int_0^{\pi} -\frac{1}{2}\left(e^{-4}-1\right)d\theta = -\frac{1}{2}\int_0^{\pi}\left(e^{-4}-1\right)d\theta = -\frac{\pi}{2}\left(e^{-4}-1\right)$$

b)

$$\int_0^1\int_y^{\sqrt{2-y^2}} (x+y)\,dx\,dy$$

$$y\le x\le\sqrt{2-y^2} \rightarrow y=\sqrt{2-y^2}$$

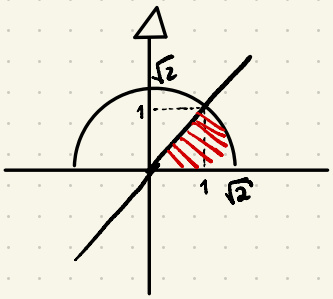
$$y^2=2-y^2$$

$$2y^2=2$$

$$y^2=1$$

$$y \rightarrow +1,\ -1$$

$$0\le y\le 1$$



$$\operatorname{sen}\theta=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}\rightarrow\theta=\frac{\pi}{4}$$

$$0\le r\le\sqrt{2}$$

$$0\le\theta\le\pi/4$$

$$\int_0^{\pi/4}\int_0^{\sqrt{2}} (r\cos\theta + r\,\mathrm{sen}\,\theta)\, r\,dr\,d\theta$$

$$= \int_0^{\pi/4}\int_0^{\sqrt{2}} (r^2\cos\theta + r^2\,\mathrm{sen}\,\theta)\,dr\,d\theta$$

$$= \int_0^{\pi/4} \left[\frac{r^3}{3}\cos\theta + \frac{r^3}{3}\,\mathrm{sen}\,\theta\right]_{r=0}^{r=\sqrt{2}} d\theta = \int_0^{\pi/4} \left(\frac{2\sqrt{2}}{3}\cos\theta + \frac{2\sqrt{2}}{3}\,\mathrm{sen}\,\theta\right) d\theta$$

$$= \frac{2\sqrt{2}}{3}\,\mathrm{sen}\,\theta\Big|_0^{\pi/4} - \frac{2\sqrt{2}}{3}\cos\theta\Big|_0^{\pi/4} = \frac{2\sqrt{2}}{3}\cdot\left(\frac{\sqrt{2}}{2} - 0\right) - \frac{2\sqrt{2}}{3}\left(\frac{\sqrt{2}}{2} - 1\right)$$

$$= \frac{2-2}{3} + \frac{2\sqrt{2}}{3} = \boxed{\frac{2\sqrt{2}}{3}}$$

2) a)

(a) $\displaystyle\int_0^{\sqrt{\pi}}\int_0^{x}\int_0^{xz} x^2\,\mathrm{sen}(y)\,dy\,dz\,dx.$

$$\int_0^{\sqrt{\pi}}\int_0^{x} -x^2\cos(y)\Big|_0^{xz}\,dz\,dx = -\int_0^{\sqrt{\pi}}\int_0^{x} x^2\cos(xz) - x^2\,dz\,dx$$

$$= -\int_0^{\sqrt{\pi}} x^2\left(\frac{\mathrm{sen}(xz)}{x}\Big|_0^{x} - x^2 z\Big|_0^{x}\right) dx = -\int_0^{\sqrt{\pi}} x\,\mathrm{sen}(xz)\Big|_0^{x} - x^3\,dx$$

$$= -\int_0^{\sqrt{\pi}} x\cdot\mathrm{sen}(x^2)\,dx \qquad u = x^2 \qquad \frac{du}{2} = x\,dx \qquad \text{y} \qquad \int_0^{\sqrt{\pi}} x^3\,dx = \frac{x^4}{4} = \frac{\pi^2}{4}$$

$$-\int_0^{\pi} \mathrm{sen}(u)\frac{du}{2} = -\left(-\cos(u)\Big|_0^{\pi}\right) = -\frac{\left(-(-1)-(-1)\right)}{2} = \frac{-2}{2} = -1 \qquad \text{entonces}$$

$$\boxed{\frac{\pi^2}{4} - 1}$$

(b) $\iiint_E e^{z/y}\,dV$ donde $E = \{(x,y,z) : 1 \le y \le 4, y \le z \le 4, 0 \le x \le z\}$

$$\int_1^4 \int_y^4 \int_0^z e^{z/y}\, dx\, dz\, dy$$

$$\int_1^4 \int_y^4 e^{z/y}\,(z)\, dz\, dy \rightarrow \qquad u = z \qquad dv = e^{z/y}$$

$$du = dz \qquad v = e^{z/y} \cdot y$$

$$\int u\,dv = z \cdot e^{z/y} \cdot y - \int e^{z/y} \cdot y\, dz$$

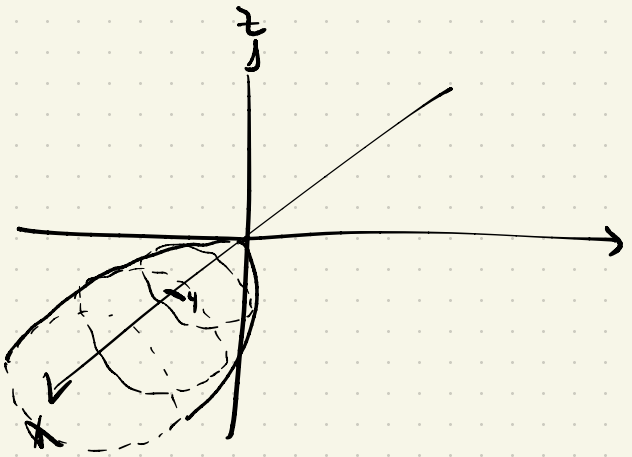
$$= z \cdot e^{z/y} \cdot y - y \cdot (e^{z/y}) \cdot y$$

$$= z \cdot e^{z/y} \cdot y - y^2 (e^{z/y})$$

$$= \int_1^4 \left(4 \cdot e^{4/y} \cdot y - y \cdot e^{4/y} \cdot y\right) - \left(y \cdot e^1 \cdot y - y^2 \cdot e^1\right) dy$$

$$= \int_1^4 4 \cdot e^{4/y} \cdot y - y^2 \cdot e^{4/y}\, dy \rightarrow \text{INconclusa.}$$

(c) $\iiint_E x\,dV$, donde E es el sólido que está acotado por el paraboloide $x = 4y^2 + 4z^2$ y el plano $x = 4$.



$y = r\cos\theta$

$z = r\,\text{sen}\,\theta$

$x = x$

$$4r^2 \le x \le 4$$

$$0 \le r \le 1$$

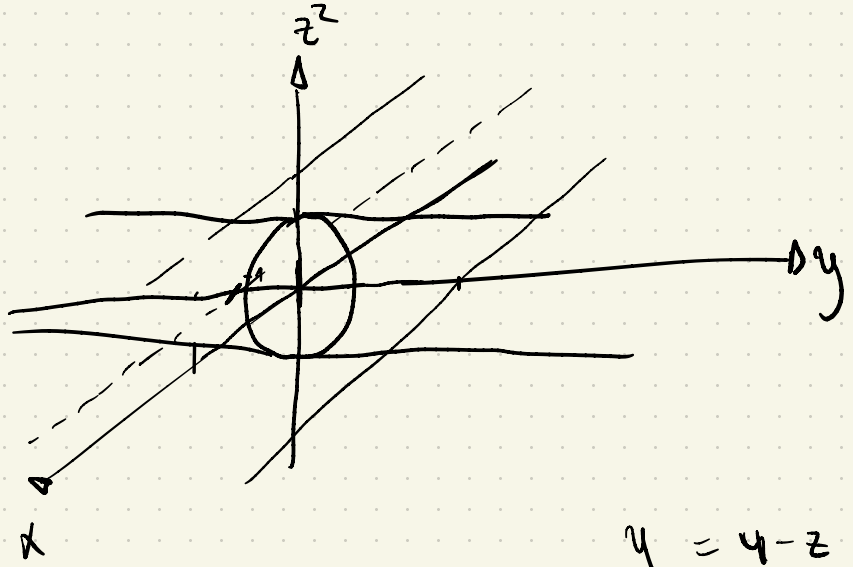
$$0 \le \theta \le 2\pi$$

$$\int_0^{2\pi}\int_0^1\int_{4r^2}^4 x\,r\,dx\,dr\,d\theta = \int_0^{2\pi}\int_0^1 \frac{x^2}{2}\Big|_{4r^2}^4 \cdot r\,dr\,d\theta$$

$$= \int_0^{2\pi}\int_0^1 \left(\frac{16r}{2} - \frac{16r^5}{2}\right) dr\,d\theta = \int_0^{2\pi}\left(4r^2 - \frac{8r^6}{6}\Big|_0^1\right) d\theta = \int_0^{2\pi}\left(4 - \frac{8}{6}(1)\right) d\theta$$

$$= 2\pi\cdot\left(4 - \frac{4}{3}\right) = 2\pi\cdot\left(\frac{12-4}{3}\right) = \boxed{\frac{16\pi}{3}}$$

3. Use integrales triples para calcular el volumen del sólido encerrado por el cilindro $x^2 + z^2 = 4$ y los planos $y = -1$ y $z + y = 4$.



$y = 4 - z$

$y = 4 - r\sin\theta$

$x = r\cos\theta$

$z = r\,\text{sen}\,\theta$

$$\int_0^{2\pi}\int_0^{2}\int_{-1}^{4-r\sin\theta} r\,dy\,dr\,d\theta \rightarrow \int_0^{2\pi}\int_0^{2} r(4 - r\sin\theta + 1)\,dr\,d\theta$$

$$\int_0^{2\pi}\left(2r^2 - \frac{r^3}{3}\sin\theta + \frac{r^2}{2}\Big|_{r=0}^{r=2}\right)d\theta$$

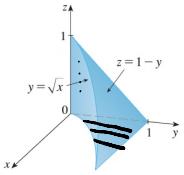
$$\int_0^{2\pi}\left(8 - \frac{8}{3}\sin\theta + \frac{4}{2}\right)d\theta = 8\cdot 2\pi + 2\cdot 2\pi - \frac{8}{3}\left(-\cos\theta\Big|_0^{2\pi}\right)$$

$$20\pi + \frac{8}{3}(1-0) = \boxed{20\pi}$$

4. La figura muestra la región de integración para la integral

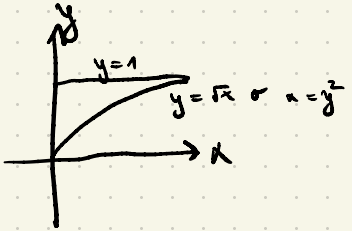
$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)\,dz\,dy\,dx$$

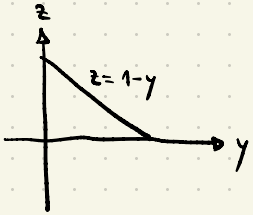
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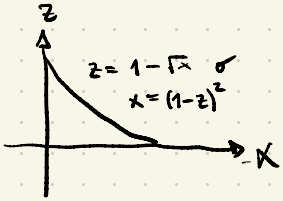


$y = 1 - z$

$dy\,dz\,dx$ ① , $dy\,dx\,dz$ ② , $dz\,dx\,dy$ ③ , $dx\,dz\,dy$ ④ , $dx\,dy\,dz$ ⑤







1.- $\int_0^1 \int_0^{1-\sqrt{x}} \int_{\sqrt{x}}^{1-z} f(x,y,z)\,dy\,dz\,dx$

2.- $\int_0^1 \int_0^{(1-z)^2} \int_{\sqrt{x}}^{1-z} f(x,y,z)\,dy\,dx\,dz$

3.- $\int_0^1 \int_0^{y^2} \int_0^{1-y} f(x,y,z)\,dz\,dx\,dy$

4. $\int_0^1 \int_0^{1-y} \int_0^{y^2} f(x,y,z)\, dx\, dz\, dy$

5. $\int_0^1 \int_0^{1-z} \int_0^{y^2} f(x,y,z)\, dx\, dy\, dz$