1.a) $\int_1^4 \int_0^2 6x^2y - 2x \, dy \, dx = \int_1^4 \left[3x^2y^2 - 2xy\right]_{y=0}^{y=2} dx$

$= \int_1^4 12x^2 - 4x \, dx$

$= \left[4x^3 - 2x^2\right]_{x=1}^{x=4}$

$= 256 - 32 - 4 + 2 = 222$

1.b) $\int_{-3}^3 \int_0^{\pi/2} y + y^2 \cos x \, dx \, dy = \int_{-3}^3 \left[yx + y^2 \operatorname{sen} x\right]_{x=0}^{x=\pi/2} dy$

$= \int_{-3}^3 \frac{\pi}{2} y + y^2 \, dy$

$= \left[\frac{\pi}{4} y^2 + \frac{1}{3} y^3\right]_{y=-3}^{y=3}$

$= \frac{\pi}{4} 9 + \frac{1}{3} 27 - \frac{\pi}{4} 9 - \frac{1}{3}(-27$

$= \frac{1}{3} 27 + \frac{1}{3} 27 = 18$

1.c) $\int_0^1 \int_0^1 xy\sqrt{x^2+y^2} \, dx \, dy = \frac{1}{2} \int_0^1 \int_{x^2}^{x^2+1} x\sqrt{u} \, du \, dx$

$u = x^2 + y^2$
$du = 2y$

$= \frac{1}{2} \int_0^1 x \left[\frac{2}{3} u^{3/2}\right]_{u=x^2}^{u=x^2+1}$

$= \frac{1}{2} \int_0^1 x \left(\frac{2}{3}(x^2+1)^{3/2} - \frac{2}{3} x^3\right)$

$= \frac{1}{2} \int_0^1 \frac{2}{3} x(x^2+1)^{3/2} - \frac{2}{3} x^4 \, dx$

$= \frac{1}{3} \left(\underbrace{\int_0^1 x(x^2+1)^{3/2}}_{I} - \underbrace{\int_0^1 x^4 dx}_{1/5}\right)$

$= \frac{1}{3}\left(\frac{1}{5}(\sqrt{32} - 1) - \frac{1}{5}\right)$

$= \frac{1}{15}(4\sqrt{2} - 2)$

$= \frac{2}{15}(2\sqrt{2} - 1)$

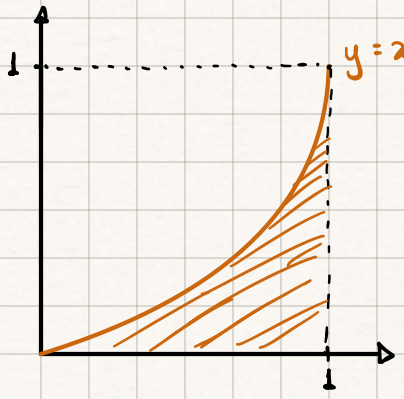
$I = \int_0^1 x(x^2+1)^{3/2} dx = \frac{1}{2} \int_1^2 K^{3/2} dK$

$K = x^2 + 1$
$dK = 2x dx$

$= \frac{1}{2}\left[\frac{2}{5} K^{5/2}\right]_{K=1}^{K=2}$

$= \frac{1}{5}(\sqrt{32} - 1)$

2. a) Dibujamos la region



$0 \le x \le 1$

$0 \le y \le x^2$

Luego

$$\iint_D x\cos y \, dA = \int_0^1 \int_0^{x^2} x\cos y \, dy\, dx$$

$$= \int_0^1 x \left[\operatorname{sen} y\right]_{y=0}^{y=x^2} dx$$

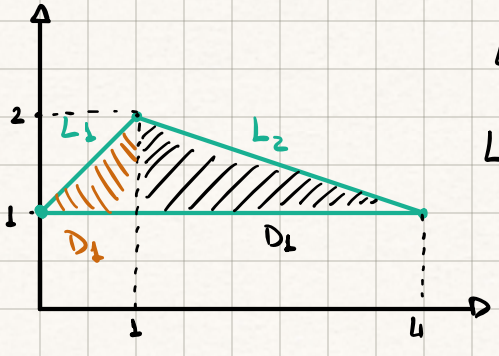
$$= \int_0^1 x \operatorname{sen} x^2 \, dx \qquad u = x^2,\ du = 2x\,dx$$

$$= \frac{1}{2}\int_0^1 \operatorname{sen} u \, du$$

$$= \frac{1}{2}\left[\cos u\right]_{u=0}^{u=1}$$

$$= \frac{1}{2}(1 - \cos 1)$$

2. b) Es necesario separar la region en 2



$L_1: y = 1 + x$

$L_2: y = \frac{7}{3} - \frac{1}{3}x$

Calculamos

·) $$\iint_{D_1} y^2 dA = \int_0^1 \int_1^{1+x} y^2 dy\, dx$$

$$= \int_0^1 \left[\frac{1}{3}y^3\right]_{y=1}^{y=1+x} dx$$

$$= \frac{1}{3}\int_0^1 (1+x)^3 - 1 \, dx \qquad u = 1+x,\ du = dx$$

$$= \frac{1}{3}\int_1^2 u^3 - 1 \, du$$

$$= \frac{1}{3}\left[\frac{1}{4}u^4 - u\right]_{u=1}^{u=2}$$

$$= \frac{1}{3}\left(\frac{16}{4} - 2 - \frac{1}{4} + 1\right)$$

$$= 1 - \frac{1}{12}$$

•) $\iint_{D_2} y^2\,dA = \int_1^4 \int_1^{\frac{7}{3}-\frac{1}{3}x} y^2\,dy\,dx$

$$= \int_1^4 \left[\frac{1}{3}y^3\right]_{y=1}^{y=\frac{7}{3}-\frac{1}{3}x} dx$$

$$= \frac{1}{3}\int_1^4 \left(\frac{7}{3} - \frac{1}{3}x\right)^3 - 1\,dx$$

$u = \frac{7}{3} - \frac{1}{3}x$
$du = -\frac{1}{3}dx$

$$= -\int_2^1 u^3 - 1\,du$$

$$= \int_1^2 u^3 - 1\,du$$

$$= \left[\frac{1}{4}u^4 - u\right]_{u=1}^{u=2}$$

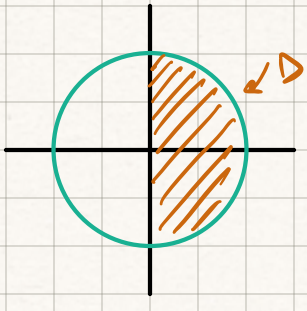
$$= \left(\frac{16}{4} - 2 - \frac{1}{4} + 1\right)$$

$$= 3 - \frac{1}{4}.$$

Luego

$$\iint_D y^2\,dA = \iint_{D_1} y^2\,dA + \iint_{D_2} y^2\,dA = 1 - \frac{1}{12} + 3 - \frac{1}{4} = 4 - \frac{1}{3}$$

2. c) Dibujamos



$$x = \sqrt{1-y^2}$$

$$x^2 + y^2 = 1.$$

Notamos que la función es simetrica para ambos cuarto de circulo, por lo que calcularemos la integral de solo la mitad superior.

$$\iint_{D^*} xy^2\,dA = \int_0^1 \int_0^{\sqrt{1-y^2}} xy^2\,dx\,dy$$

$$= \int_0^1 y^2 \int_0^{\sqrt{1-y^2}} x\,dx\,dy$$

$$= \int_0^1 y^2 \left[\frac{1}{2}x^2\right]_{x=0}^{x=\sqrt{1-y^2}}$$

$$= \frac{1}{2}\int_0^1 y^2(1-y^2)\,dy$$

$$= \frac{1}{2}\int_0^1 y^2 - y^4$$

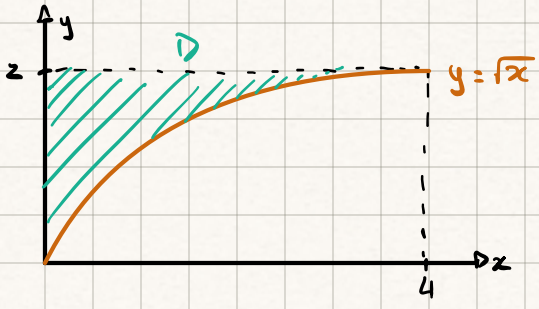
$$= \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right)$$

$$= \frac{1}{15}$$

luego

$$\iint_D xy^2\,dA = 2\iint_{D^*} xy^2\,dA = \frac{2}{15}$$

3) Tenemos que $0 \le x \le 4$, $\sqrt{x} \le y \le 2$



$0 \le x \le y^2$

$0 \le y \le 2$

Luego

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\,dy\,dx = \int_0^2 \int_0^{y^2} \frac{1}{y^3+1}\,dx\,dy$$

$$= \int_0^2 \left[\frac{1}{y^3+1}x\right]_{x=0}^{x=y^2} dy$$

$$= \int_0^2 \frac{1}{y^3+1}y^2\,dy \qquad u = y^3+1,\ du = 2y^2\,dy$$

$$= \frac{1}{2}\int_1^9 \frac{1}{u}\,du$$

$$= \frac{1}{2}\left[\log u\right]_{u=1}^{u=9}$$

$$= \frac{1}{2}\log 9$$