PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
FACULTAD DE MATEMATICAS
DEPARTAMENTO DE MATEMATICA
Segundo semestre 2023

# Ayudantía 13- MAT1620

1. Calcule las siguientes integrales iteradas

   (a) $\int_1^4 \int_0^2 (6x^2y - 2x)dydx$.

   (b) $\int_{-3}^3 \int_0^{\pi/2} (y + y^2\cos(x))dxdy$.

   (c) $\int_0^1 \int_0^1 xy\sqrt{x^2+y^2}dydx$.

2. Evalúe las siguientes integrales.

   (a) $\iint_D x\cos(y)dA$, donde $D$ está acotada por $y = 0$, $y = x^2$ y $x = 1$.

   (b) $\iint_D y^2dA$, donde $D$ es la región triangular con vértices $(0, 1), (1, 2)$ y $(4, 1)$.

   (c) $\iint_D xy^2dA$, donde $D$ está acotada por $x = 0$ y $x = \sqrt{1-y^2}$.

3. Determine el volumen del sólido encerrado por el cilindro parabólico $y = x^2$ y los planos $z = 3y$, $z = 2 + y$.

4. Invierta el orden de integración para calcualr el valor de

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dydx$$

1. Calcule las siguientes integrales iteradas

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a) $\int_1^4 \left( \frac{6x^2}{2}y^2 - 2xy \right)\Big|_0^2 dx$

$= \int_1^4 \left(3x^2(4) - 2x\cdot(2)\right) dx$

$= 4x^3 - 2x^2 \Big|_1^4 = 4(4^3 - 1^3) - 2(4^2 - 1^2)$

$= 4\cdot 63 - 2\cdot(15)$

$= 252 - 30$

$= 222$

$$\int_{-3}^{3}\int_{0}^{\pi/2}(y+y^2\cos(x))\,dx\,dy = \int_{-3}^{3}\left(yx+y^2\operatorname{sen}(x)\Big|_0^{\pi/2}\right)dy$$

$$= \int_{-3}^{3}\left(y(\pi/2)+y^2\left(\operatorname{sen}(\pi/2)-\operatorname{sen}(0)\right)\right)dy$$

$$= \int_{-3}^{3}\left(y\cdot\frac{\pi}{2}+y^2\right)dy = \frac{\pi}{2}\cdot\frac{y^2}{2}\Big|_{-3}^{3} + \frac{y^3}{3}\Big|_{-3}^{3}$$

$$= \frac{\pi}{2}\cdot\left(\frac{9-9}{2}\right)+\frac{1}{3}\cdot(27-(-27))$$

$$= \frac{\pi}{2}\cdot(0)+\frac{1}{3}\cdot(54) = \boxed{18}$$

c) $\int_0^1\int_0^1 xy\sqrt{x^2+y^2}\,dy\,dx$

$u = x^2 \qquad v = y^2$

$\frac{du}{dx} = 2x \qquad \frac{dv}{dy} = 2y$

$\int_0^1\int_0^1 \sqrt{u+v}$

$\int_0^1\int_0^1 \frac{1}{4}\sqrt{u+v}\,dv\,du$

$$\frac{1}{4}\int_0^1\left((u+v)^{3/2}\cdot\frac{2}{3}\Big|_0^1\right)du = \frac{2}{12}\int_0^1\left((u+1)^{3/2}-(u)^{3/2}\right)du$$

$$= \frac{2}{12} \int_0^1 \left( (u+1)^{3/2} - u^{3/2} \right) du$$

$$= \frac{2}{12} \left( (u+1)^{5/2} \cdot \frac{2}{5} - u^{5/2} \cdot \frac{2}{5} \right) \Big|_0^1$$

$$= \frac{2}{12} \left( \left( (1+1)^{5/2} - 1^{5/2} \right) \cdot \frac{2}{5} - \left( 1 \cdot \frac{2}{5} \right) \right)$$

$$= \frac{2}{12} \cdot \left( \left( 2^{5/2} - 1 \right) \cdot \frac{2}{5} - \frac{2}{5} \right)$$

2, 4, 8, 16, 32

$$= \frac{2}{15} \left( 2\sqrt{2} - 1 \right)$$

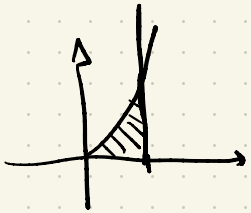
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a) $\iint_D x\cos(y)\,dA$



$0 \le x \le 1$

$0 \le y \le x^2$

$\int_0^1 \int_0^{x^2} x\cos(y)\,dy\,dx$

$\rightarrow \int_0^1 x \cdot \operatorname{sen}(y)\Big|_0^{x^2} dx$

$\int_0^1 x \cdot \operatorname{sen}(x^2)\,dx$

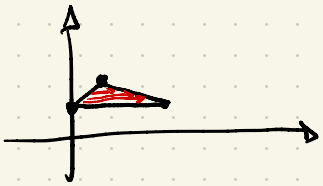
$u = x^2$

$\frac{du}{dx} = 2x \rightarrow \frac{du}{2} = dx$

$\int_0^1 \operatorname{sen}(u)\frac{du}{2} = -\frac{\cos(u)}{2}\Big|_0^1$

$= -\frac{1}{2}\left(\cos(1) - \cos(0)\right) = -\frac{1}{2}\left(\cos(1) - 1\right)$

b) $\iint_D y^2 \, dA$ $(0,1), (1,2), (4,1)$



$$\int_1^2 \int_{y-1}^{7-3y} y^2 \, dx\, dy$$

$y = 1 + x$

$x = y - 1$

$$\int_1^2 \left( \left( (7-3y) - (y-1) \right) y \right) dy$$

$m = \frac{2-1}{1-4} = \frac{1}{-3}$

$y - 2 = \frac{-1}{3}(x-1)$

$y = -\frac{1}{3} + \frac{1+6}{3} = -\frac{1}{3}x + \frac{7}{3}$

$$\int_1^2 (8y - 4y^2) \, dy$$

$3y = -x + 7$

$x = 7 - 3y$

$$4y^2 - \frac{4}{3}y^3 \Big|_1^2 = 4(4-1) - \frac{4}{3}(8-1)$$

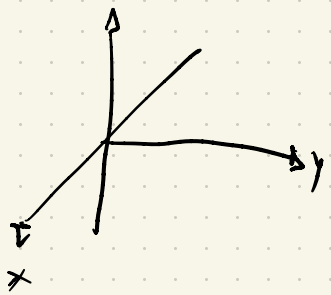
$$= 4 \cdot 3 - \frac{4}{3}(7)$$

$$= 12 - \frac{28}{3}$$

Volumen encerrado por los cilindros $z = x^2$, $y = x^2$ y planos $z = 0$, $y = 4$

Recordar que

$$V = \iiint_D dV \rightarrow \int\int\int$$



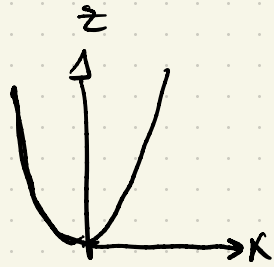

Si $z = 0$

$\downarrow$

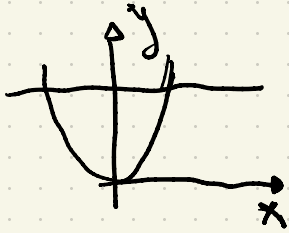
$0 = x^2 \rightarrow x = 0$

$$x^2 \leq y \leq 4$$

$$0 \leq z \leq x^2$$




Si $y = 4 \rightarrow 4 = x^2 \rightarrow x \rightarrow 2, -2$




Entonces

$$\int_{-2}^{2}\int_{0}^{x^2}\int_{x^2}^{4} dy\,dz\,dx = \int_{-2}^{2}\int_{0}^{x^2} (4 - x^2)\,dz\,dx$$

$$= \int_{-2}^{2} -(x^4 - 4x^2)\,dx = -\left(\frac{x^5}{5} - \frac{4}{3}x^3\Big|_{-2}^{2}\right) = -\left(\frac{2^5 - (-2)^5}{5} - \frac{4}{3}\left(2^3 - (-2)^3\right)\right)$$

$$= -\left(\frac{64}{5} - \frac{4}{3}\cdot(16)\right) = -\left(-\frac{2\cdot 64}{15}\right) = +\frac{128}{15}$$

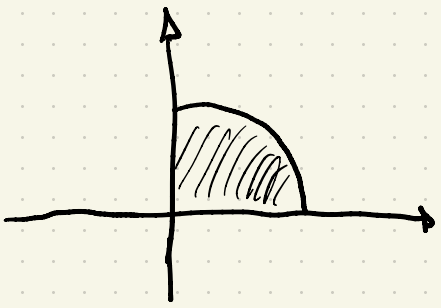
$$\iint_D xy^2\,dA$$

acotada por

$x=0$ y $x=\sqrt{1-y^2}$

$$x^2 = 1-y^2$$

$$x^2+y^2=1$$



$$\int_0^1 \int_0^{\sqrt{1-y^2}} xy^2\,dx\,dy = \int_0^1 \frac{x^2}{2}y^2 \Big|_0^{\sqrt{1-y^2}}\,dy$$

$$= \int_0^1 \frac{1-y^2}{2}\cdot y^2\,dy$$

$$= \frac{1}{2}\left(\frac{y^3}{3}-\frac{y^5}{5}\right)\Big|_0^1 = \frac{1}{2}\left(\frac{1-0}{3}-\frac{1-0}{5}\right)$$

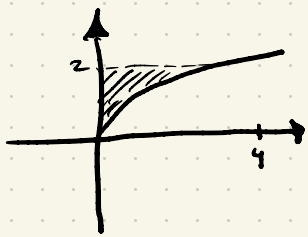
$$= \frac{1}{2}\left(\frac{5-3}{15}\right) = \frac{1}{15}$$

4. Invierta el orden de integración para calcualr el valor de

$$\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dydx$$

$\sqrt{x} \leq y \leq 2$

$0 \leq x \leq 4$




$0 \leq x \leq y^2$

$0 \leq y \leq 2$

$$\int_0^2 \int_0^{y^2} \frac{1}{y^3+1}\, dxdy = \int_0^2 \frac{y^2}{y^3+1}\, dy$$

$u = y^3 \qquad \frac{du}{dy} = 3y^2 \rightarrow \frac{du}{3} = y^2\, dy$

$$\int_0^8 \frac{1}{u+1}\frac{du}{3} \rightarrow \frac{1}{3}\ln|u+1|\Big|_0^8 = \frac{1}{3}(\ln(9) - \ln(1))$$

$$= \frac{1}{3}\ln(9)$$