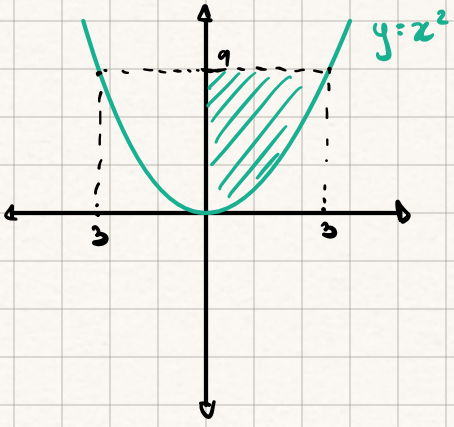
1.a) Nos conviene hacer una inversión de región, ya que $\int \cos y^2 \, dy$ no es sencillo de resolver. Trabajemos




$$0 \le y \le 9$$

$$0 \le x \le \sqrt{y}$$

$$\int_0^3 \int_{x^2}^9 x \cos y^2 \, dy\,dx = \int_0^9 \int_0^{\sqrt{y}} x \cos y^2 \, dx\,dy$$

$$= \int_0^9 \left[ \frac{1}{2} x^2 \cos y^2 \right]_{x=0}^{x=\sqrt{y}}$$

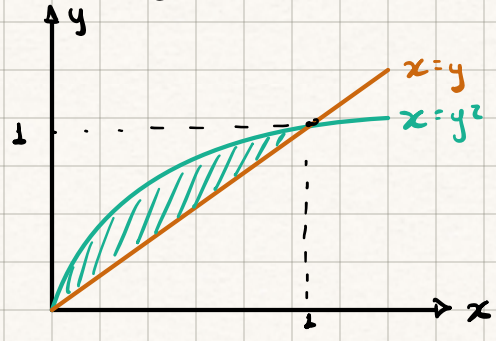
$$= \int_0^9 \frac{1}{2} y \cos y^2 \, dy \qquad u = y^2, \; du = 2y\,dy$$

$$= \frac{1}{4} \int_0^{81} \cos u \, dy$$

$$= \frac{1}{4} \left[ \operatorname{sen} u \right]_{u=0}^{u=81}$$

$$= \frac{1}{4} \operatorname{sen} 81$$

1.b) Las superficies $x = y^2$ y $x = y$ nos delimitan un area en el plano xy




$$0 \le y \le 1$$

$$y^2 \le x \le y$$

Luego calculamos el volumen debajo de la superficie $x^2+y^2$ en el area

$$\int_0^1 \int_{y^2}^{y} x^2 + y^2 \, dx\,dy = \int_0^1 \left[ \frac{1}{3} x^3 + xy^2 \right]_{x=y^2}^{x=y} dy$$

$$= \int_0^1 \frac{1}{3} y^3 + y^3 - \frac{1}{3} y^6 - y^4 \, dy$$

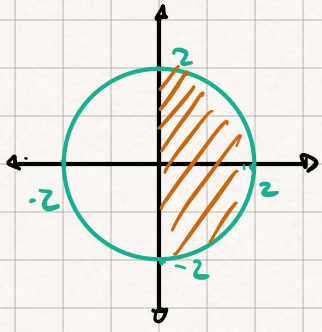
$$= \int_0^1 \frac{4}{3} y^3 - y^4 - \frac{1}{3} y^6 \, dy$$

$$= \frac{4}{3}\cdot\frac{1}{4} - \frac{1}{5} - \frac{1}{3}\cdot\frac{1}{7}$$

$$= \frac{1}{3} - \frac{1}{5} - \frac{1}{21}$$

$$= \frac{9}{105}$$

2.a) reescribiendo $x = \sqrt{4-y^2}$ tenemos que $x^2+y^2=4$, el cual es el cirwlo de radio 2. luego




$0 \le r \le 2$ $\qquad x = r\cos\theta$

$\frac{-\pi}{2} \le \theta \le \frac{\pi}{2}$ $\qquad y = r\operatorname{sen}\theta$

Luego

$$\iint_D e^{-(x^2+y^2)}\,dA = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}}\int_0^2 e^{-r^2} r\,dr\,d\theta \qquad u = r^2,\ du = 2r\,dr$$

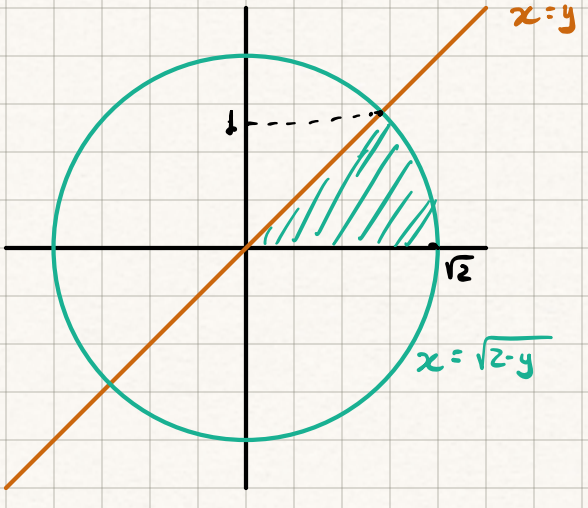
$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2}\int_0^4 e^{-u}\,du\,d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2}\left[-e^{-u}\right]_{u=0}^{u=4} d\theta$$

$$= \frac{1}{2}\int_{-\pi/2}^{\pi/2} 1 - e^{-4}\,d\theta$$

$$= \frac{\pi}{2}\left(1-e^{-4}\right)$$

2.b) Graficamos la region



en coordenadas polares $r \in [0, \sqrt{2}]$ y $\theta$ va desde 0 hasta $\frac{\pi}{4}$ y a que en este angulo $x = y \Rightarrow \cos\theta = \operatorname{sen}\theta$. Luego

$$\begin{aligned}
\int_0^1 \int_y^{\sqrt{2-y^2}} x+y \, dx\, dy &= \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{2}} (r\cos\theta + r\operatorname{sen}\theta) r \, dr\, d\theta \\
&= \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{2}} r^2(\cos\theta + \operatorname{sen}\theta)\, dr\, d\theta \\
&= \int_0^{\frac{\pi}{4}} \cos\theta + \operatorname{sen}\theta \int_0^{\sqrt{2}} r^2 \, dr\, d\theta \\
&= \int_0^{\frac{\pi}{4}} (\cos\theta + \operatorname{sen}\theta) \left[\tfrac{1}{3} r^3\right]_{r=0}^{r=\sqrt{2}} d\theta \\
&= \frac{2\sqrt{2}}{3} \int_0^{\frac{\pi}{4}} \cos\theta - \operatorname{sen}\theta \, d\theta \\
&= \frac{2\sqrt{2}}{3} \left[\operatorname{sen}\theta - \cos\theta\right]_{\theta=0}^{\theta=\pi/4} \\
&= \frac{2\sqrt{2}}{3} \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} - (0-1)\right) \\
&= \frac{2\sqrt{2}}{3}
\end{aligned}$$

3) Queremos integrar la Funcion $z = \sqrt{x^2+y^2}$ en la region del cirwlo de radio 2. hacemos coordenadas polares. Luego

$$\begin{aligned}
V &= \int_0^2 \int_0^{2\pi} \sqrt{r^2} \cdot r \, d\theta\, dr \\
&= \int_0^2 \int_0^{2\pi} r^2 \, d\theta\, dr \\
&= 2\pi \int_0^2 r^2 dr \\
&= 2\pi \cdot \frac{8}{3} \\
&= \frac{16\pi}{3}.
\end{aligned}$$