Ayudantía 10

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Problema 1

Aplique la regla de la cadena para hallar dz/dt.

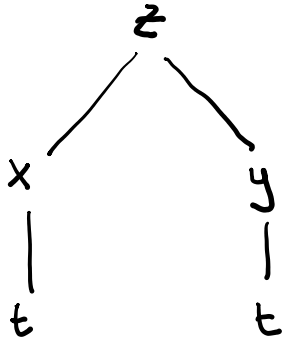
1)

$$z = x^2 + y^2 + xy, \qquad x = \sin t, \qquad y = \exp t$$

2)

$$z = \sqrt{x^2 + y^2 + 1}, \qquad x = \ln t, \qquad y = \cos t$$

usamos un diagrama de arbol



$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

1) $z = x^2 + y^2 + xy \qquad x = \sin(t) \qquad y = e^t$

$$\frac{dz}{dt} = (2x+y)\cos(t) + (2y+x)e^t$$

$$\frac{dz}{dt} = (2\sin(t) + e^t)\cos(t) + (2e^t + \sin(t))e^t$$

2) $z = \sqrt{x^2+y^2+1} \qquad x = \ln(t) \qquad y = \cos(t)$

$$\frac{dz}{dt} = \frac{1}{2}(x^2+y^2+1)^{-1/2} \cdot 2x \cdot \frac{1}{t} + \frac{1}{2}(x^2+y^2+1)^{-1/2} \cdot 2y \cdot -\sin(t)$$

$$= x(x^2+y^2+1)^{-1/2} \cdot \frac{1}{t} - y(x^2+y^2+1)^{-1/2} \cdot \sin(t)$$

$$\frac{dz}{dt} = \frac{\ln(t)}{t}(\ln^2(t) + \cos^2(t) + 1)^{-1/2} - \cos(t)\sin(t)(\ln^2(t) + \cos^2(t) + 1)^{-1/2}$$

Problema 2

Mediante la regla de la cadena encuentre $\partial z/\partial s$ y $\partial z/\partial t$.

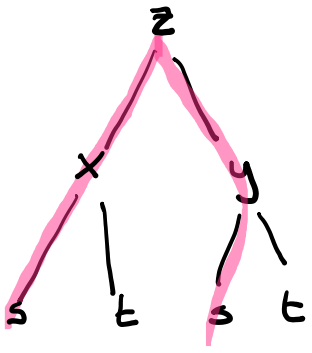
1)

$$z = x^2y^3, \qquad x = s\cos t, \qquad y = s\sin t$$

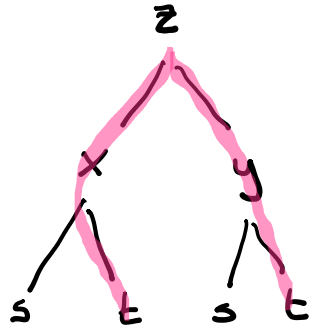
2)

$$z = \exp x + 2y, \qquad x = s/t, \qquad y = t/s$$

usamos un diagrama de arbol



$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$



$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

1) $z = x^2y^3 \qquad x = s\cos(t) \qquad y = s\cdot\sin(t)$

$$\frac{\partial z}{\partial s} = 2xy^3(\cos(t)) + 3x^2y^2(\sin(t))$$

$$= 2s^4\cos^2(t)\sin^3(t) + 3s^4\cos^2(t)\sin^3(t)$$

$$\frac{\partial z}{\partial s} = 5s^4\cos^2(t)\sin^3(t)$$

$$\frac{\partial z}{\partial t} = 2xy^3(-s\cdot\sin(t)) + 3x^2y^2(s\cos(t))$$

$$\frac{\partial z}{\partial t} = -2s^5\cos(t)\sin^4(t) + 3s^5\sin^2(t)\cos^4(t)$$

2) $z = e^x + 2y \qquad x = \frac{s}{t} \qquad y = \frac{t}{s}$

$$\frac{\partial z}{\partial s} = e^x \cdot \frac{1}{t} + 2 \cdot \frac{-t}{s^2}$$

$$\frac{\partial z}{\partial s} = e^{s/t} \cdot \frac{1}{t} - 2\frac{t}{s^2}$$

$$\frac{\partial z}{\partial t} = e^x \cdot \frac{-s}{t^2} + 2 \cdot \frac{1}{s}$$

$$\frac{\partial z}{\partial t} = -e^{s/t} \cdot \frac{s}{t^2} + \frac{2}{s}$$

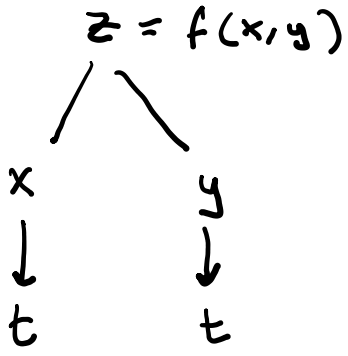
Problema 3

Si $z = f(x, y)$, donde f es derivable,

$$x = g(t) \qquad y = h(t)$$
$$g(3) = 2 \qquad h(3) = 7$$
$$g'(3) = 5 \qquad h'(3) = -4$$
$$f_x(2, 7) = 6 \qquad f_y(2, 7) = -8$$

determine dz/dt cuando $t = 3$.

Usamos un diagrama de arbol



entonces

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = f_x \cdot \frac{dx}{dt} + f_y \cdot \frac{dy}{dt}$$

para $t = 3$ $\quad x = 2$ $\quad y = 7$

$x' = 5$ $\quad y' = -4$

Ademas que $\quad f_x(2,7) = 6$ $\quad f_y(2,7) = -8$

así, simplemente $\quad \frac{dz}{dt} = 6 \cdot 5 + (-8)(-4) = 62$

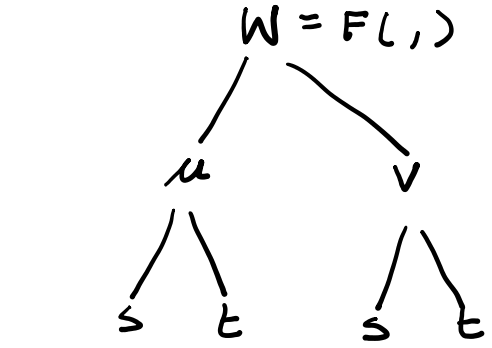
Problema 4

Sea $W(s,t) = F(u(s,t), v(s,t))$, donde F, u y v son derivables,

$$u(1,0) = 2 \qquad v(1,0) = 3$$
$$u_s(1,0) = -2 \qquad v_s(1,0) = 5$$
$$u_t(1,0) = 6 \qquad v_t(1,0) = 4$$
$$F_u(2,3) = -1 \qquad F_v(2,3) = 10$$

Determine $W_s(1,0)$ y $W_t(1,0)$.

Usamos un diagrama de arbol

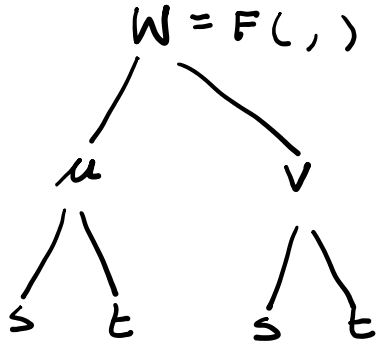


$$\frac{\partial W}{\partial s} = \frac{\partial W}{\partial u}\frac{\partial u}{\partial s} + \frac{\partial W}{\partial v}\frac{\partial v}{\partial s}$$

$$= F_u \cdot u_s + F_v \cdot v_s$$

$$W_s(1,0) = (-1)\cdot(-2) + 10 \cdot 5$$

$$= 52$$



$$\frac{\partial W}{\partial t} = \frac{\partial W}{\partial u}\frac{\partial u}{\partial t} + \frac{\partial W}{\partial v}\frac{\partial v}{\partial t}$$

$$= F_u \cdot u_t + F_v \cdot v_t$$

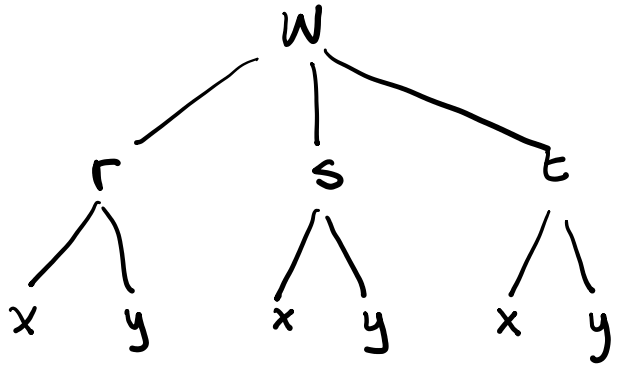
$$W_t(1,0) = (-1)\cdot 6 + 10 \cdot 4$$

$$= 34$$

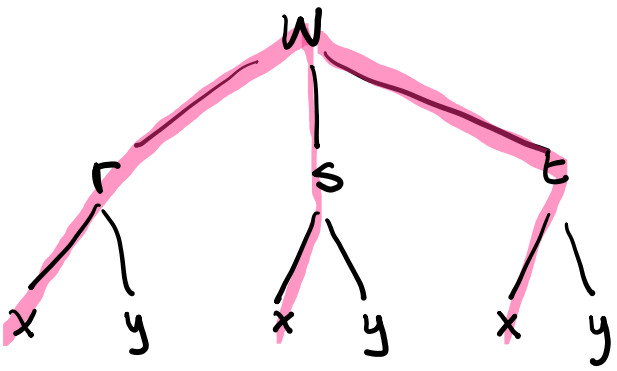
Problema 5

Mediante un diagrama de árbol, escriba la regla de la cadena para el caso dado. Suponga que todas las funciones son derivables.

1) w= f(r,s,t), donde r= r(x,y), s=s(x,y), t=t(x,y)

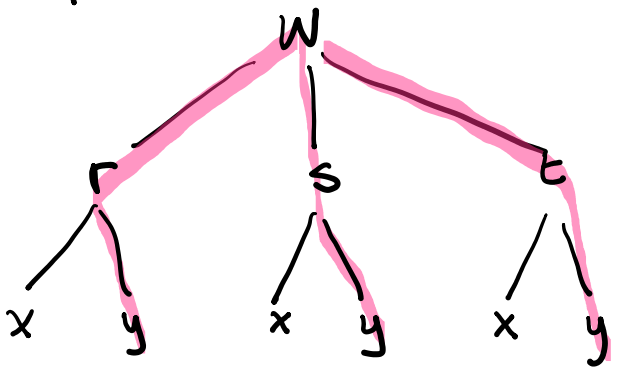


para $\frac{\partial w}{\partial x}$



$$\frac{\partial W}{\partial x} = \frac{\partial W}{\partial r}\frac{\partial r}{\partial x} + \frac{\partial W}{\partial s}\frac{\partial s}{\partial x} + \frac{\partial W}{\partial t}\frac{\partial t}{\partial x}$$

para $\frac{\partial w}{\partial y}$



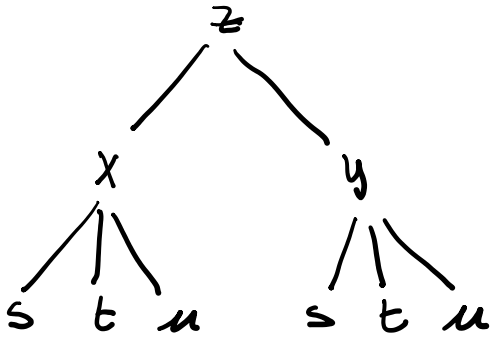
$$\frac{\partial W}{\partial y} = \frac{\partial W}{\partial r}\frac{\partial r}{\partial y} + \frac{\partial W}{\partial s}\frac{\partial s}{\partial y} + \frac{\partial W}{\partial t}\frac{\partial t}{\partial y}$$

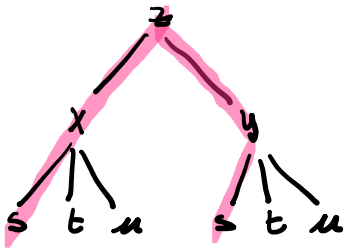
Problema 6

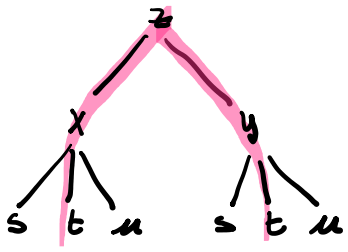
Use la regla de la cadena para calcular por separado las derivadas parciales de z con respecto a cada una de las variables independientes (no las intermedias).

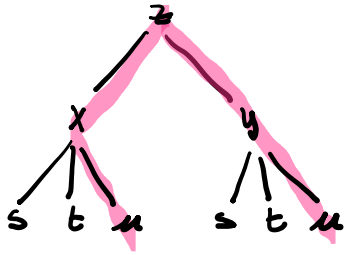
$$z - x^4 + x^2y, \quad x - s + 2t - u, \quad y - stu^2$$

Hacemos un diagrama de arbol









para $\frac{\partial z}{\partial s}$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}$$

para $\frac{\partial z}{\partial t}$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

para $\frac{\partial z}{\partial u}$

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}$$

$$\frac{\partial z}{\partial s} = (4x^3 + 2xy)(1) + x^2(tu^2)$$
$$= 4(s+2t-u)^3 + 2stu^2(s+2t-u) + (s+2t-u)tu^2$$

$$\frac{\partial z}{\partial t} = (4x^3 + 2xy)(2) + x^2(su^2)$$
$$= 2(4(s+2t-u)^3 + 2(s+2t-u)stu^2) + (s+2t-u)^2 su^2$$

$$\frac{\partial z}{\partial u} = (4x^3 + 2xy)(-1) + x^2(2stu)$$
$$= -(4(s+2t-u)^3 + 2stu^2(s+2t-u)) + (s+2t-u)^2(2stu)$$