PONTIFICIA UNIVERSIDAD CATOLICA DE CHILE
FACULTAD DE MATEMATICAS
DEPARTAMENTO DE MATEMATICA
Segundo semestre 2023

Ayudantía 9- MAT1620

1. Aplique la regla de la cadena para encontrar $\dfrac{dz}{dt}$

   (a) $z = x^2 + y^2 + xy, x = \text{sen}(t), y = e^t.$

   (b) $z = xe^{y/z}, x = t^2, z = 1 + 2t.$

2. Aplique la regla de la cadena para encontrar $\dfrac{\partial z}{\partial s}$ y $\dfrac{\partial z}{\partial t}$

   (a) $z = \text{arcsen}(x - y), x = s^2 + t^2, y1 - 2st.$

   (b) $e^r \cos(\theta), r = st, \theta = \sqrt{s^2 + t^2}.$

3. Suponga que f es una función derivable en variables x e y, y que

$$g(u, v) = f(e^u + \text{sen}(v), e^u + \cos(v))$$

   Use la tabla de valores para calcular $g_u(0, 0)$ y $g_v(0, 0)$.

	f	g	f_x	f_y
(0, 0)	3	6	4	8
(1, 2)	6	3	2	5

4. Demuestre que cualquier función de la forma

$$z = f(x + at) + g(x - at)$$

   es una solución de la ecuación diferencial

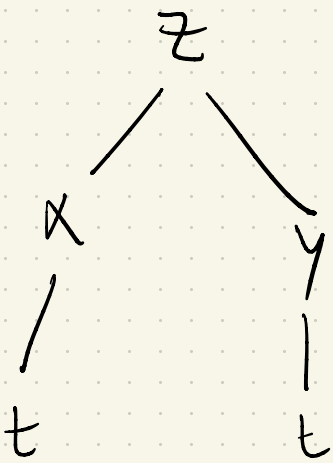
$$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$$

1. Aplique la regla de la cadena para encontrar $\frac{dz}{dt}$

   (a) $z = x^2 + y^2 + xy, x = \text{sen}(t), y = e^t.$

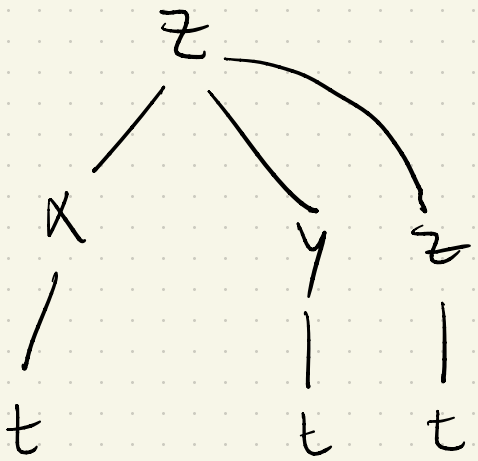
   (b) $z = xe^{y/z}, x = t^2, z = 1 + 2t.$

a)



$$\frac{dz}{dt} = \frac{dz}{dx}\cdot\frac{dx}{dt} + \frac{dz}{dy}\cdot\frac{dy}{dt}$$

$$= (2x+y)\cdot\cos(t) + (2y+x)\cdot e^t$$



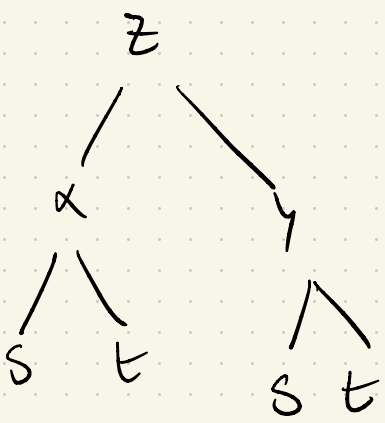
$$\frac{dz}{dt} = \frac{dz}{dx}\cdot\frac{dx}{dt} + \frac{dz}{dy}\cdot\frac{dy}{dt} + dz$$

$$e^{x/y}\cdot 2t + xe^{y/z}\cdot 0 + xe^{y/z}\cdot\frac{-1}{z^2}\cdot 2$$

2. Aplique la regla de la cadena para encontrar $\frac{\partial z}{\partial s}$ y $\frac{\partial z}{\partial t}$

(a) $z = \operatorname{arcsen}(x - y), x = s^2 + t^2, y 1 - 2st.$

(b) $e^r \cos(\theta), r = st, \theta = \sqrt{s^2 + t^2}.$



$f(x) = \operatorname{arcsen}(u)'$

$\downarrow$

$- \frac{u'}{\sqrt{1-u^2}}$

$$\frac{\partial z}{\partial s} = -\frac{1}{\sqrt{1-(x-y)^2}} \cdot 1 \cdot (2s) + \frac{-1 \cdot -1}{\sqrt{1-(x-y)^2}} \cdot (-2t)$$

$$\frac{\partial z}{\partial s} = \frac{-2s}{\sqrt{1-(x-y)^2}} - \frac{2t}{\sqrt{1-(x-y)^2}}$$

$$\frac{\partial z}{\partial t} = \frac{-2t}{\sqrt{1-(x-y)^2}} - \frac{2s}{\sqrt{1-(x-y)^2}}$$

b) $\frac{\partial z}{\partial s} = e^{v}\cos\theta \cdot t + -e^{v}\operatorname{sen}(\theta) \cdot \frac{1}{2}\frac{1}{\sqrt{s^2+t^2}} \cdot 2s$

$\frac{\partial z}{\partial t} = e^{v}\cos\theta \cdot s + -e^{v}\operatorname{sen}(\theta) \cdot \frac{1}{2}\frac{1}{\sqrt{s^2+t^2}} \cdot 2t$

3. Suponga que f es una función derivable en variables x e y, y que

$$g(u,v) = f(e^u + \text{sen}(v), e^u + \cos(v))$$

Use la tabla de valores para calcular $g_u(0,0)$ y $g_v(0,0)$.

	f	g	f_x	f_y
$(0,0)$	3	6	4	8
$(1,2)$	6	3	2	5

$$f(x(u,v), y(u,v)) = g(u,v)$$

$$\frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$f_x(1\,2)\cdot(e^u) + f_y(1,2)\cdot e^u = g_u(0,0)$$

$$\rightarrow \quad 2\cdot 1 + 5\cdot 1 = g_u(0,0) = \boxed{7}$$

$$g_v(0,0) = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial v} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial v} = f_x(1,2)\cdot\cos(v) - f_y(1,2)\cdot\text{sen}(v)$$

$$= 2\cdot 1 + 5\cdot 0 = 2$$

4. Demuestre que cualquier función de la forma

$$z = f(x + at) + g(x - at)$$

es una solución de la ecuación diferencial

$$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$$

$$\frac{\partial z}{\partial t} = f'(x+at) \cdot a + g'(x-at) \cdot -a$$

$$\frac{\partial^2 z}{\partial t^2} = f''(x+at) \cdot a^2 + g''(x-at) \cdot a^2$$

$$\frac{\partial z}{\partial x} = f'(x+at) + g'(x-at)$$

$$\frac{\partial^2 z}{\partial x^2} = f''(x+at) + g''(x-at)$$