Ayudantía 8

1.- Plano que pasa por ptos

$(1,2,3)$, $(-1,0,1)$ y $(0,8,-1)$

A B C

$\vec{AB} = (-2,-2,-2)$

$\vec{AC} = (-1,6,-4)$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & -2 & -2 \\ -1 & 6 & -4 \end{vmatrix}$$

$$= \langle 20, -(8-2), -12-2 \rangle$$

$$\langle 20, -6, -14 \rangle$$

EC. plano.

$$\begin{pmatrix} x-1 \\ y-2 \\ z-3 \end{pmatrix} \cdot \begin{pmatrix} 20 \\ -6 \\ -14 \end{pmatrix} = 0$$

$$\rightarrow \quad 20x - 6y - 14z = 20 - 12 - 42$$

$$\boxed{20x - 6y - 14z = -36}$$

0

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2)

$$\lim_{x,y \to 0,0} \frac{x^2 + 3xy + y^2}{3x^2 - xy + 3y^2}$$

transformando

$$\lim_{r \to 0} \frac{r^2 \cos^2\theta + 3r^2 \operatorname{sen}\theta \cos\theta + r^2 \operatorname{sen}^2\theta}{3r^2 \cos^2\theta - r^2 \cos\theta \operatorname{sen}\theta + 3r^2 \operatorname{sen}^2\theta}$$

$$\lim_{r \to 0} \frac{r^2 + 3r^2 \operatorname{sen}\theta \cos\theta}{3r^2 - r^2 \cos\theta \operatorname{sen}\theta}$$

$$\lim_{r \to 0} \frac{r^2(1 + 3 \operatorname{sen}\theta \cos\theta)}{r^2(3 - 1 \cdot \cos\theta \operatorname{sen}\theta)}$$

$$= \frac{1 + 3 \operatorname{sen}\theta \cos\theta}{3 - \cos\theta \operatorname{sen}\theta}$$

no existe el límite.

b) $\lim_{x,y \to (0,0)} \frac{xy}{x^2+y^2} = \lim_{r \to 0} \frac{r^2 \cos\theta \operatorname{sen}\theta}{r^2}$

$= \cos\theta \operatorname{sen}\theta$

no existe.

3).- Es lo mismo que el de la ayudantía 7.

4) $f(x,y) = \sqrt{x + e^{3y}}$ diferenciable en $(3,0)$

$\frac{\partial f}{\partial x} = \frac{1}{2} \frac{1}{\sqrt{x+e^{3y}}}$

$\frac{\partial f}{\partial y} = \frac{1}{2} \frac{1}{\sqrt{x+e^{3y}}} \cdot e^{3y} \cdot 3$

y como las derivadas parciales son continuas en $(3,0)$, entonces f es diferenciable en $(3,0)$.

$$\frac{\partial f}{\partial x} = \frac{1}{2}\frac{1}{\sqrt{x+e^{3y}}}$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}\frac{1}{\sqrt{x+e^{3y}}} \cdot e^{3y} \cdot 3$$

Aproximamos $f(2{,}9,\ 0{,}1)$, entonces buscamos la aproximación lineal en un punto cercano a $(2{,}9,\ 0{,}1)$, elegimos $(3,0)$

$$\rightarrow f(x,y) \approx \underbrace{\sqrt{3+e^0}}_{f(3,0)} + \underbrace{\frac{1}{2}\cdot\frac{1}{\sqrt{1+3}}}_{\frac{\partial f}{\partial x}(3,0)} \cdot (x-3) + \underbrace{\frac{1}{2}\frac{3}{\sqrt{1+3}}}_{\frac{\partial f}{\partial y}(3,0)} y$$

$$f(2{,}9,\ 0{,}1) \approx \sqrt{4} + \frac{1}{4}\cdot(2{,}9-3) + \frac{1}{2}\cdot\frac{3}{2}\cdot 0{,}1$$

$$F(2{,}9,\ 0{,}1) \approx 2 + \frac{1}{4}\cdot -0{,}1 + \frac{3}{4}\cdot 0{,}1$$

$$F(2{,}9,\ 0{,}1) \approx 2 + \frac{1}{2}\cdot 0{,}1$$

$$F(2{,}9,\ 0{,}1) \approx 2 + 0{,}05 = 2{,}05$$

5) $f(x,y) = g(x\cos(y),\ x\sin(y))$

$$h(x,y) = \left(x\,\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$$

Equivalentemente podemos decir que

$$g(u(x,y),\ v(x,y)), \quad \begin{matrix} u = x\cos y \\ v = x\sin y \end{matrix}$$

$$\rightarrow \frac{\partial f}{\partial x} = \frac{\partial g}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial x}$$

$$\frac{\partial f}{\partial x} = \frac{\partial g}{\partial u}\cdot\cos y + \frac{\partial g}{\partial v}\cdot\sin y$$

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial g}{\partial v}\cdot\frac{\partial v}{\partial y}$$

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial u}\cdot(-x\sin y) + \frac{\partial g}{\partial v}\cdot(x\cos y)$$

$$h(x,y) = (x)^2 \cdot \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2$$

$$= x^2\left(\left(\frac{\partial g}{\partial u}\cos y\right)^2 + \left(\frac{\partial g}{\partial v}\sin y\right)^2 + 2\frac{\partial g}{\partial u}\cos y \cdot \frac{\partial g}{\partial v}\cdot \sin y\right)$$

$$+\left(x^2\left(\left(\frac{\partial g}{\partial u}\right)^2 \sin^2 y\right) + x^2\left(\left(\frac{\partial g}{\partial v}\right)^2 \cos^2 y\right) - 2x^2\frac{\partial g}{\partial u}\cdot\frac{\partial g}{\partial v}\cdot \cos y \cdot \sin(y)\right)$$

$$= x^2 \cdot \left(\frac{\partial g}{\partial u}\right)^2 (\cos^2 y + \sin^2 y) + x^2\left(\frac{\partial g}{\partial v}\right)^2 (\sin^2 y + \cos^2 y)$$

$$\rightarrow x^2\left(\left(\frac{\partial g}{\partial u}\right)^2 + \left(\frac{\partial g}{\partial v}\right)^2\right)$$

y como las derivadas parciales de g son constantes entonces

$h(x,y)$ no depende de y.