① a) (F) $P[u]_{B_1} = [u]_{B_2}$

$$\begin{pmatrix}1\\1\end{pmatrix} = \alpha\begin{pmatrix}2\\1\end{pmatrix} + \beta\begin{pmatrix}2\\3\end{pmatrix}$$

$$\begin{pmatrix}1\\2\end{pmatrix} = \gamma\begin{pmatrix}2\\1\end{pmatrix} + \delta\begin{pmatrix}2\\3\end{pmatrix}$$

con $P = \begin{bmatrix}\alpha & \gamma\\ \beta & \delta\end{bmatrix}$

si $P = \begin{bmatrix}3 & 1\\ -1 & 1\end{bmatrix} \rightarrow \begin{pmatrix}1\\1\end{pmatrix} \neq 3\begin{pmatrix}2\\1\end{pmatrix} - \begin{pmatrix}2\\3\end{pmatrix}$

$$\neq \begin{pmatrix}4\\0\end{pmatrix}$$

b) (F) $\mathrm{Im}(T) = \left\{ \begin{pmatrix} a-c\\ c-a\\ b+d\end{pmatrix} \right\}$

$$= \mathrm{Gen}\left\{ \begin{pmatrix}1\\-1\\0\end{pmatrix}\begin{pmatrix}-1\\1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\right\}$$

$$= \mathrm{Gen}\left\{ \begin{pmatrix}1\\-1\\0\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}\right\} \neq \mathbb{R}^3$$

∴ no es sobreyectiva

pues ej. $\begin{pmatrix}0\\1\\0\end{pmatrix} \notin \mathrm{Im}(T)$

(2) a) (V) $A=\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & 4 \end{bmatrix}$ $\det(A-xI)=0$

$(2-x)\left[(1-x)(4-x)+2\right]=0$

$(2-x)(x^2-5x+6)=0$

$(2-x)(x-2)(x-3)=0$

$E_3 = \text{Nul}\begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 2 \\ 0 & -1 & 1 \end{bmatrix} = \text{Gen}\left\{\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}\right\}$

$E_2 = \text{Nul}\begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & -1 & 2 \end{bmatrix} = \text{Gen}\left\{\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \\ 1 \end{pmatrix}\right\}$

$\sum mg = 1+2=3$ Sí es diagonalizable

b) (F) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ no es diagonalizable

$E_1 = \text{Nul}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \text{Gen}\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\}$

$\sum mg = 1 \neq 2$

(3) a) (V) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1/3 \end{bmatrix}$

$E_1 = \text{Gen}\left\{\begin{pmatrix} 1 \\ 0 \end{pmatrix}\right\}$ $\quad E_{1/3} = \text{Gen}\left\{\begin{pmatrix} 3 \\ -2 \end{pmatrix}\right\}$

$$\begin{pmatrix} 14 \\ -8 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 4\begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$A^k \begin{pmatrix} 14 \\ -8 \end{pmatrix} = 2A^k \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 4A^k \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$A^k \begin{pmatrix} 14 \\ -8 \end{pmatrix} = 2 \cdot 1^k \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 4 \cdot (1/3)^k \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$\lim_{k \to \infty} A^k \begin{pmatrix} 14 \\ -8 \end{pmatrix} = 2 \cdot 1 \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} + 4 \cdot 0 \cdot \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

b) (V) Sea $\{u, v, w\}$ ortogonal

Si $\alpha u + \beta v + \gamma w = \vec{0}$

$u \cdot (\alpha u + \beta v + \gamma w) = 0$

$\alpha u \cdot u = 0$

$\alpha = 0$

$v \cdot [\alpha u + \beta v + \gamma w] = 0$

$\beta v \cdot v = 0$

$\beta = 0$

$$w \cdot (\alpha u + \beta v + \gamma w) = 0$$

$$\gamma \, w \cdot w = 0$$

$$\gamma = 0$$

$\therefore \{u, v, w\}$ es L.I.

④ a) ⑦

$$P = \begin{bmatrix} 5/6 & -2/6 & -1/6 \\ -2/6 & 2/6 & -2/6 \\ -1/6 & -2/6 & 5/6 \end{bmatrix}$$

$$\sim \begin{bmatrix} 5 & -2 & -1 \\ -2 & 2 & -2 \\ -1 & -2 & 5 \end{bmatrix} \sim \begin{bmatrix} 0 & 3 & -6 \\ 1 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\rightarrow V = \text{Gen}\left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix} \right\}$$

b) (V) $Im(A^TA) = Im(A^T)$

Si $v \in Im(A^TA) \rightarrow \exists x: A^TAx = v$

$A^T(Ax) = v$

$\rightarrow v \in Im(A^T)$

Si $v \in Im(A^T) \rightarrow \exists y: A^Ty = v$

$y = u + w$ con $u \in Col(A)$

$w \in Nul(A^T)$

entonces $u = Ax$, $A^Tw = \vec{0}$

$A^Ty = A^Tu + A^Tw$

$v = A^TAx + \vec{0}$

$\rightarrow v \in Im(A^TA)$

□