Problema 1

Considere circuito de la Figura 1 y asuma que este se encuentra operando en régimen sinusoidal permanente.

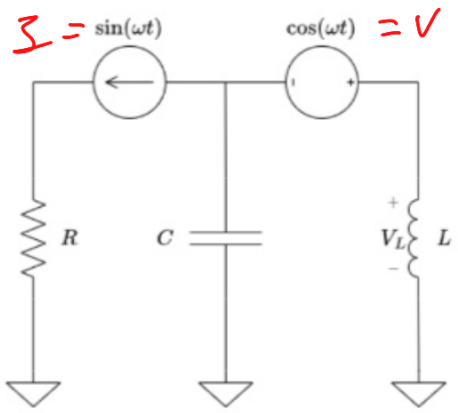


Figura 1: Circuito Problema 1.

Calcule:

1. El voltaje del inductor L de la forma $a + jb$.
2. El voltaje del inductor L de forma fasorial.
3. El voltaje del inductor L en el tiempo.

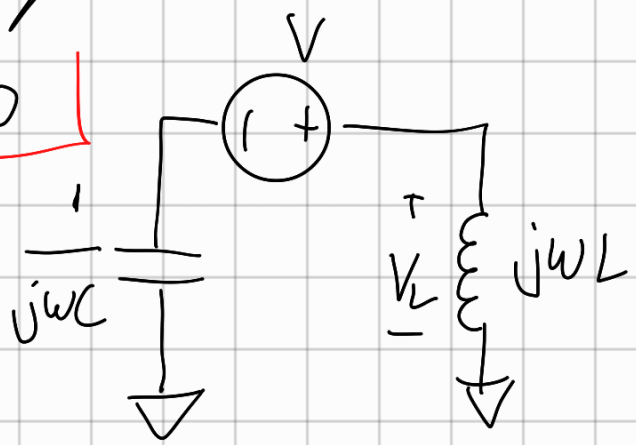
$V = 1\angle 0° = 1$

$I = \cos(\omega t - 90°)$

$I = 1\angle -90° = -j$

a) Superposición:

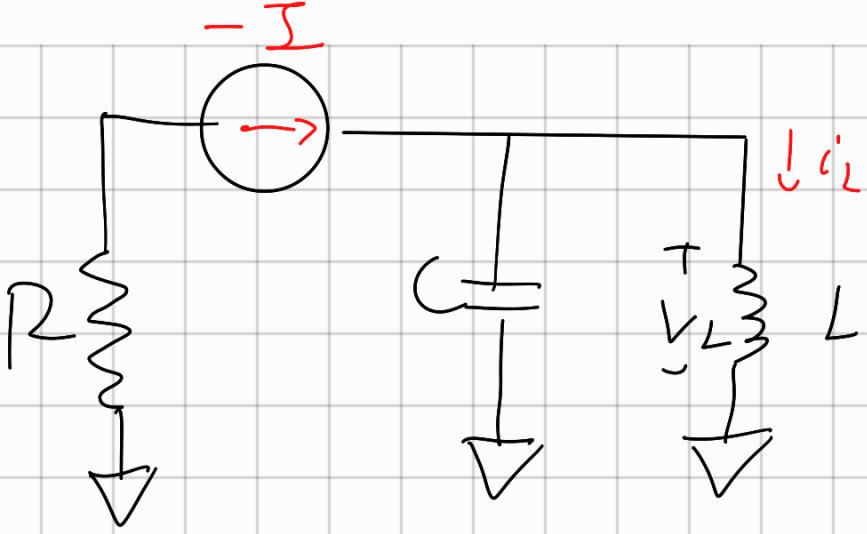
$I = 0$



$$V_L' = \frac{j\omega L}{j\omega L + \frac{1}{j\omega C}} V \cdot \frac{j\omega C}{j\omega C} = \frac{j\omega L \, j\omega C}{1 + j^2 \omega L C}$$

$$V_L' = -\frac{\omega^2 L C}{1 - \omega^2 L C}$$

$V = 0$



$$i_L = (-I)\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + j\omega L \cdot \frac{j\omega C}{j\omega C}} = j\frac{1}{1-\omega^2 LC}$$

$(-I) = +j$

$$V_L^2 = i_L \cdot j\omega L = -\frac{\omega L}{1-\omega^2 LC}$$

$$V_L = V_L^1 + V_L^2 = -\frac{\omega^2 LC}{1-\omega^2 LC} - \frac{\omega L}{1-\omega^2 LC}$$

$$\boxed{V_L = -\frac{\omega L(1+\omega C)}{1-\omega^2 LC}}$$

b)

Si $1-\omega^2 LC > 0 \Rightarrow \vec{V_L} = \frac{\omega L(1+\omega C)}{1-\omega^2 LC} \angle 180°$

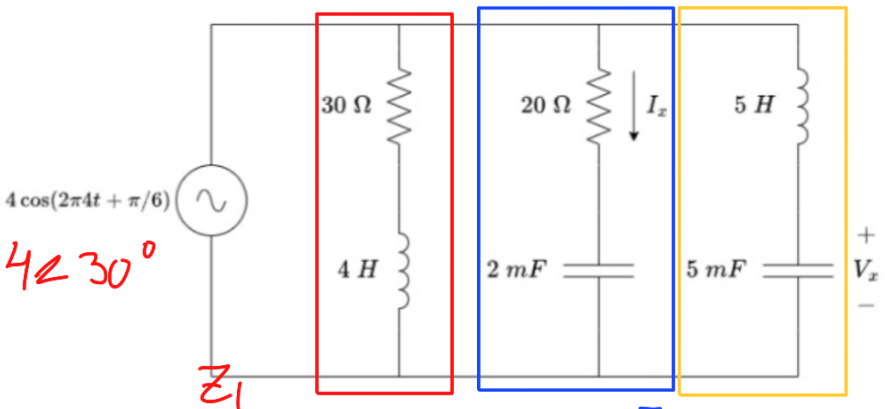
Si $1-\omega^2 LC < 0 \Rightarrow \vec{V_L} = \frac{\omega L(1+\omega C)}{\omega^2 LC - 1} \angle 0°$

c) $v_L(t) = \frac{\omega L(1+\omega C)}{|1-\omega^2 LC|}\cos(\omega t + \phi)$

$$\phi = \begin{cases} 0° & \text{si } 1-\omega^2 LC < 0 \\ 180° & \text{si } 1-\omega^2 LC > 0 \end{cases}$$

Problema 2

Considere que el circuito de la Figura 2 opera en régimen sinusoidal permanente.



$4\angle 30°$

Z_1 Z_2 Z_3

Figura 2: Circuito Problema 2.

- a) Determine la impedancia de entrada vista por la fuente de voltaje.
- b) Encuentre la corriente I_x en forma fasorial.
- c) Utilizando equivalente de Thévenin-Norton encuentre el voltaje V_x de forma fasorial.

a) $Z_{in} = Z_1 \,||\, Z_2 \,||\, Z_3$

$= (30 + j100,5) \,||\, (20 - j19,9) \,||\, j117,7$

$104,9\angle 73,4° \,||\, 28,2\angle -44,85° \,||\, 117,7\angle 90°$

$$Z_{in} = \frac{Z_1 Z_2 Z_3}{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}$$

$-6890,55 + j7299$

$$\frac{1}{Z_{in}} = \frac{348354\angle 118,53°}{10037\angle -226,65°}$$

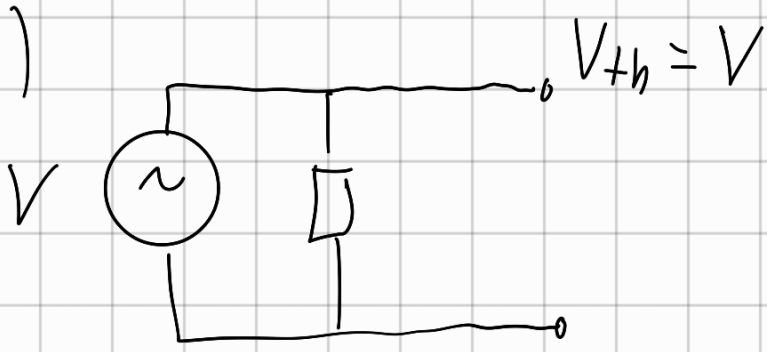
$$\frac{1}{Z_{in}} = 34,7 \angle 345,18° = 34,7 \angle -14,82°$$

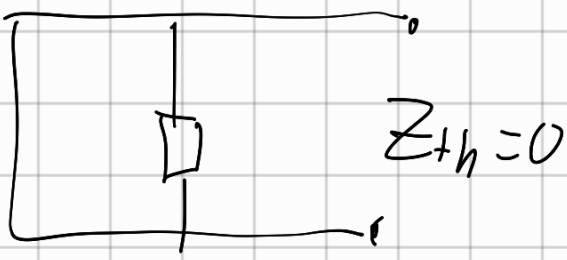
b) $I_x = \frac{V}{Z_2} \Rightarrow |Z_2| = \sqrt{20^2 + 19,9^2} = 28,2$

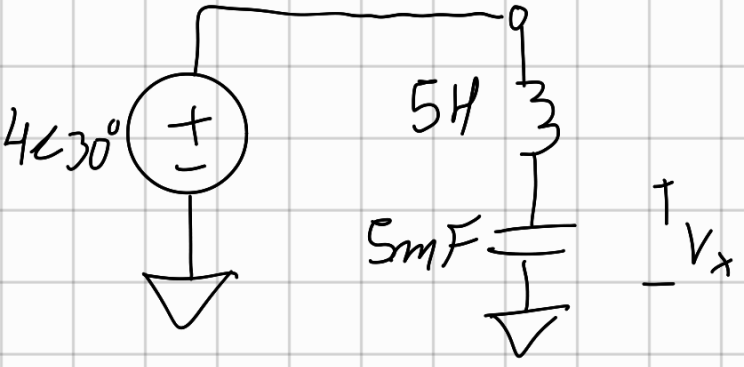
$$\angle Z_2 = \arctan\left(\frac{19,9}{20}\right) = -44,84°$$

$$\hat{I}_x = \frac{4 \angle 30°}{28,2 \angle -44,84°} = 0,142 \angle 74,84° \, [A]$$

c)







$$V_x = 4 \angle 30° \frac{\frac{1}{j2\pi 4.5 \cdot 10^{-3}}}{\frac{1}{j2\pi 4.5 \cdot 10^{-3}} + j2\pi 4.5} = -\frac{j7,957}{j117,7} 4 \angle 30°$$

$$V_x = -\frac{7,957}{117,7} 4 \angle 30° = 0,0676 \angle -180° \cdot 4 \angle 30°$$

$$V_x = 0,27 \angle -150°$$

Problema 3

Considere el circuito operando en régimen sinusoidal permanente que se muestra en la Figura 3.

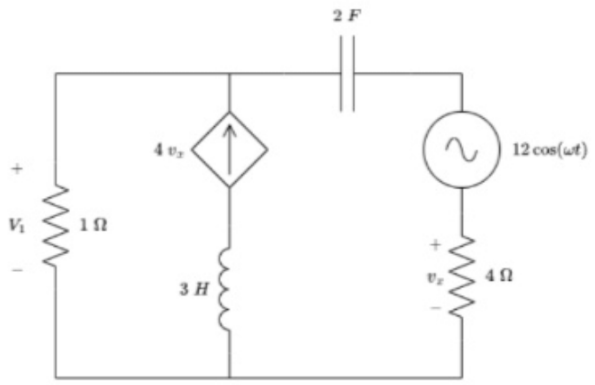


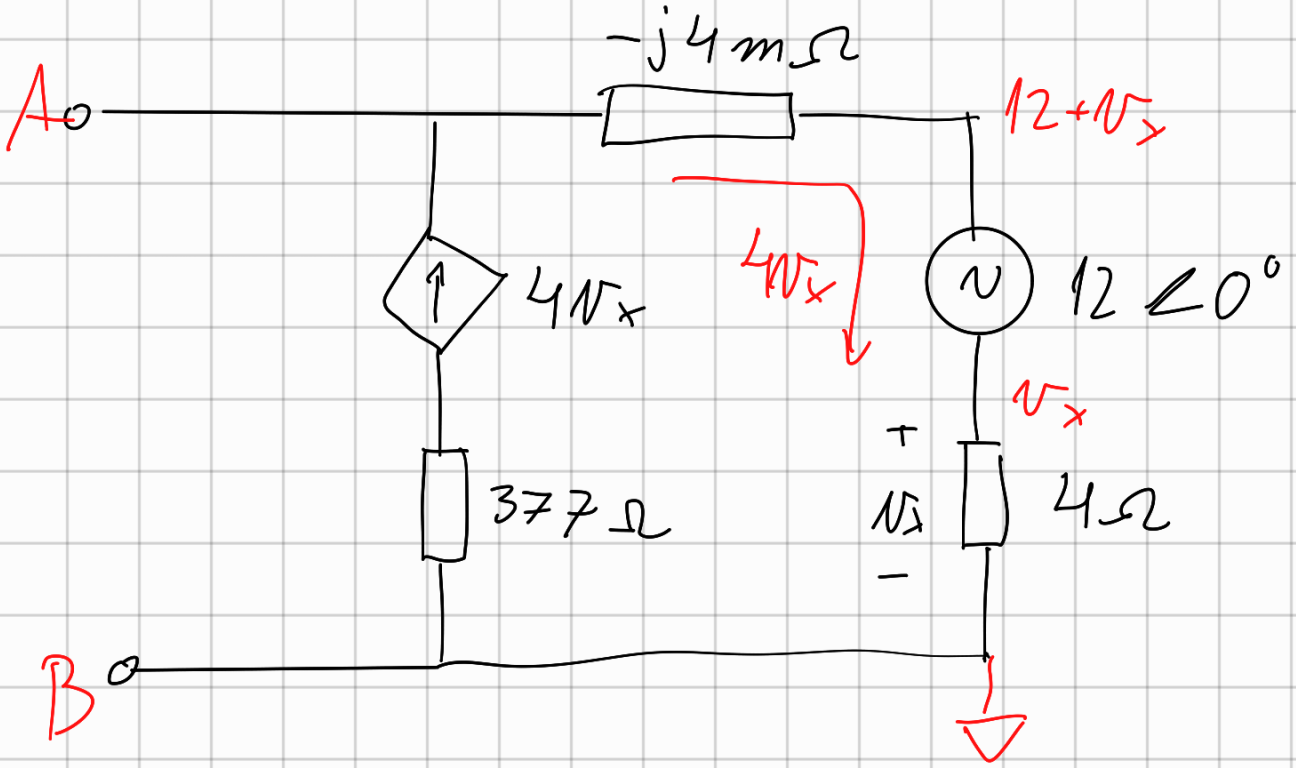
Figura 3: Circuito Problema 3.

$\omega = 2\pi \cdot 20 = 40\pi$

Si el sistema funciona con una frecuencia de $f = 20\ Hz$, que cambios se le deben hacer al circuito para que el voltaje V_1 tenga un desfase [redacted]. (*Hint: Piense en agregar resistores, capacitores y/o inductores*).

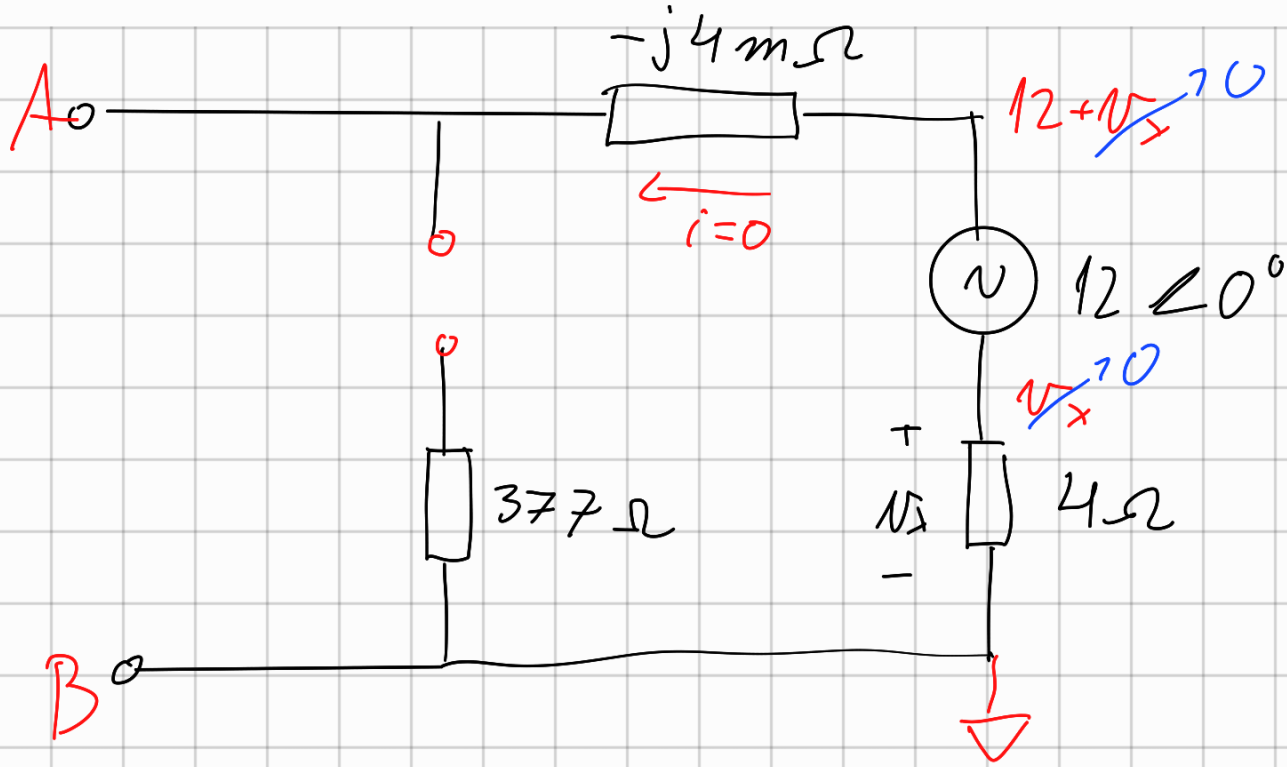
Puramente real → $\pm 180°, 0°$

Hacemos Thévenin:

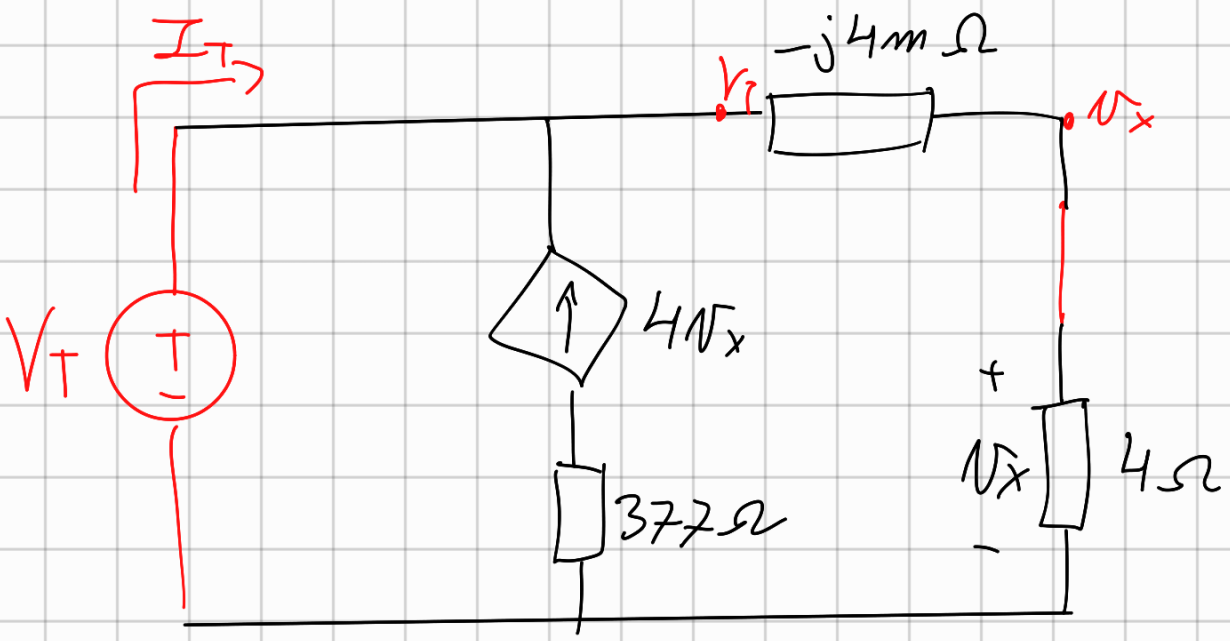


Podemos ver que $\frac{V_x}{4} = 4V_x$

$\Rightarrow V_x = 0$



$$\Rightarrow V_1 = V_{Th} = 12\angle 0^\circ$$



$$(1)\quad I_T + 4V_x = \frac{V_T - V_x}{-j4m}$$

$$(2)\quad \frac{V_T - V_x}{-j4m} = \frac{V_x}{4} \Rightarrow V_T - V_x = -jV_x m$$

$$V_T = V_x(1 - jm)$$

$$V_x = \frac{V_T}{1 - jm}$$

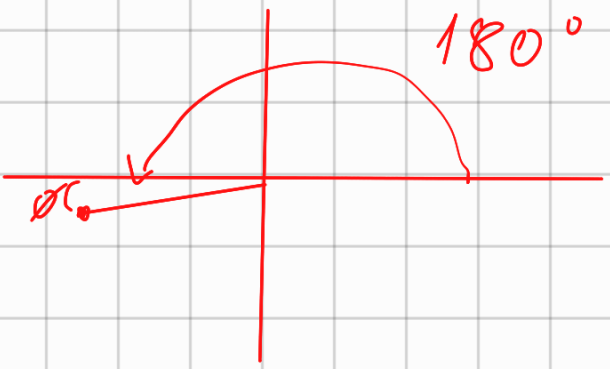
(1) (2) $I_T + \frac{4V_T}{1-jm} = \frac{V_T}{-j4m} - \frac{V_T}{-j4m(1-jm)}$

$I_T = V_T \left(\frac{-1}{1-jm} - \frac{1}{j4m} + \frac{1}{j4m(1-jm)} \right)$

$I_T = V_T \ (-0,75 - j\,0,75m)$

$\frac{V_T}{I_T} = Z_{th} = -1,33 - j1,33m$

$\Rightarrow |Z_{th}| = 1,33$

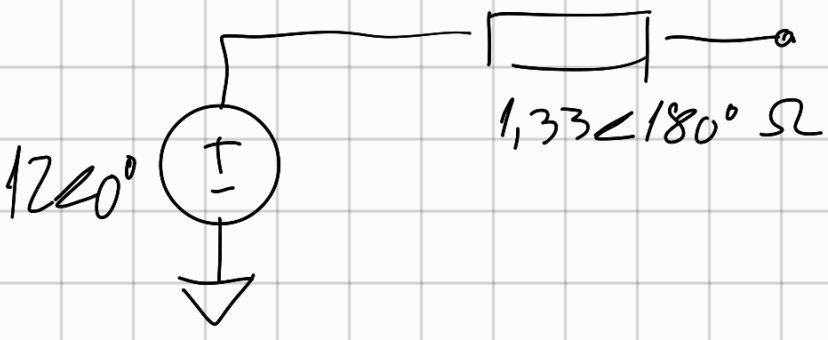


$\phi = \arctan\left(\frac{1,33m}{1,33}\right)$

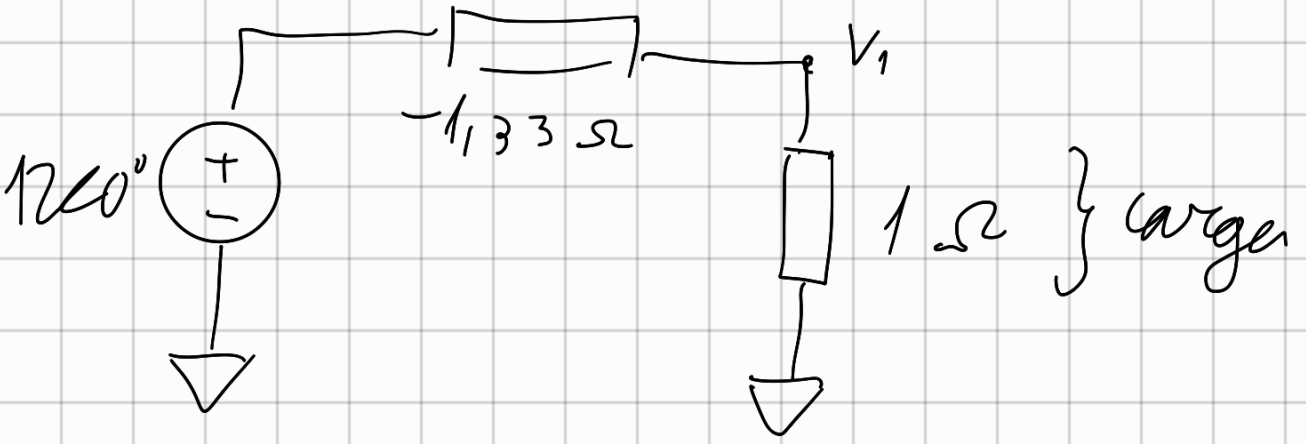
$\phi \approx 0,1°$

$\Rightarrow \angle Z_{th} = 180° + 0,1° = 180°$

$Z_{th} = 1,33 \angle 180°$



Conectamos 1 Ω:



$$V_1 = 12\angle 0° \frac{1\angle 0°}{0{,}33\angle 180°} = 36\angle -180°$$

Vemos que el desfase es de −180°.

Podemos ver que no es necesario conectar nada.

* Con el error de arrastre que se cometió en la ayudantía:

(1) $\Sigma_T + 4V_x = \frac{V_T - V_x}{-j4m}$

(2) $\frac{V_T - V_x}{-j4m} = \frac{V_x}{4} \Rightarrow V_T - V_x = -jV_x$

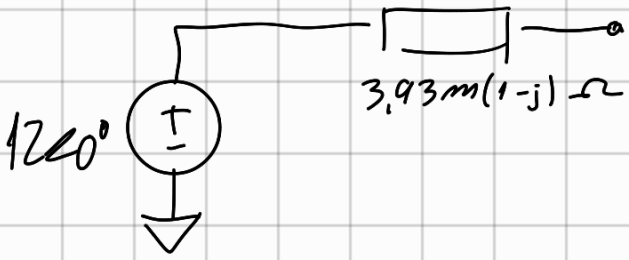
Se omitió un m

$V_T = V_x(1-j)$

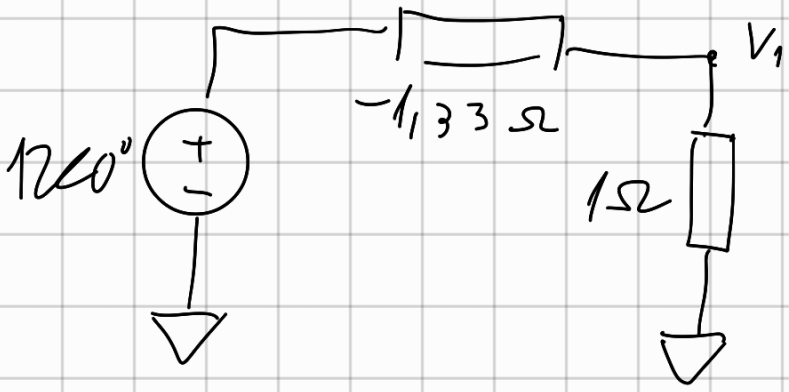
$V_x = \frac{V_T}{1-j}$

Asumamos que es $\underline{\underline{m}}$ no iba para ver que ocurre.

$$\frac{V_1}{\Sigma_1} = Z_{th} = \frac{1-j}{254} = 3{,}93m - j\,3{,}93m$$



Conectamos $1\,\Omega$:

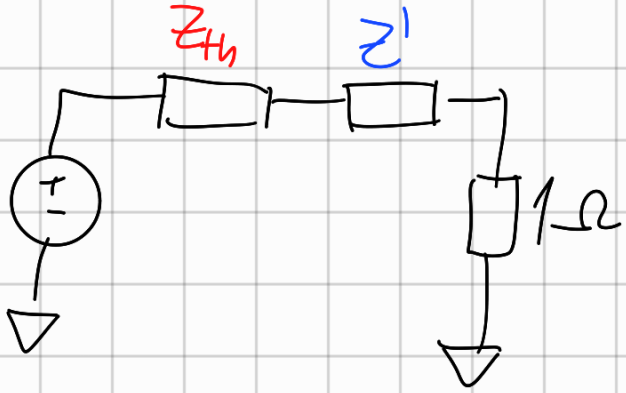


$$V_1 = 1\angle 0^\circ \frac{1\angle 0^\circ}{1{,}004\angle -0{,}22^\circ}$$

$$V_1 = 1\angle 0{,}22^\circ$$

Para que la fase sea real (0° en este caso) debemos conectar un inductor en serie:

$$V_1 = 1\angle 0^\circ \frac{1\angle 0^\circ}{Z_{th} + Z' + 1}$$



Para 0:

$$3{,}93m + 1 - j\,3{,}93m + j\,3{,}93m$$

$$jwL = j\,3{,}93m \Rightarrow L = \frac{3{,}93m}{w}$$

$$\boxed{L = 31{,}27\,\mu H}$$

* Si bien este no es el resultado correcto, se considera el error de arrastre hecho en la ayudantía para ver este caso.

Problema 4

Considere el circuito de la Figura 4.

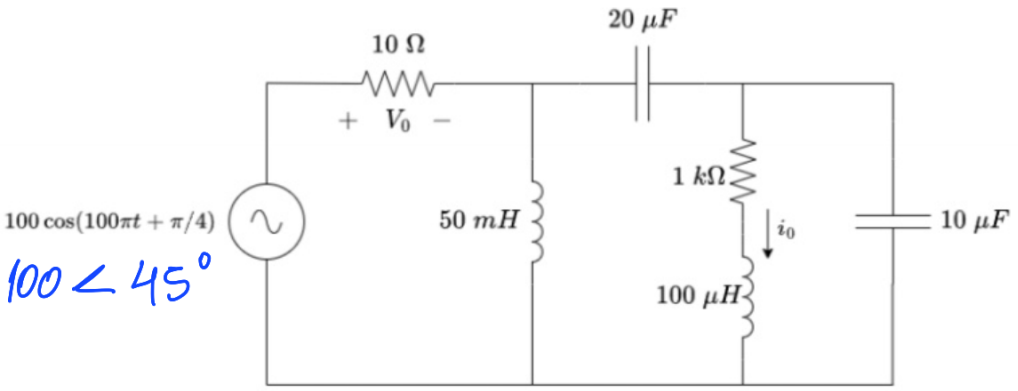
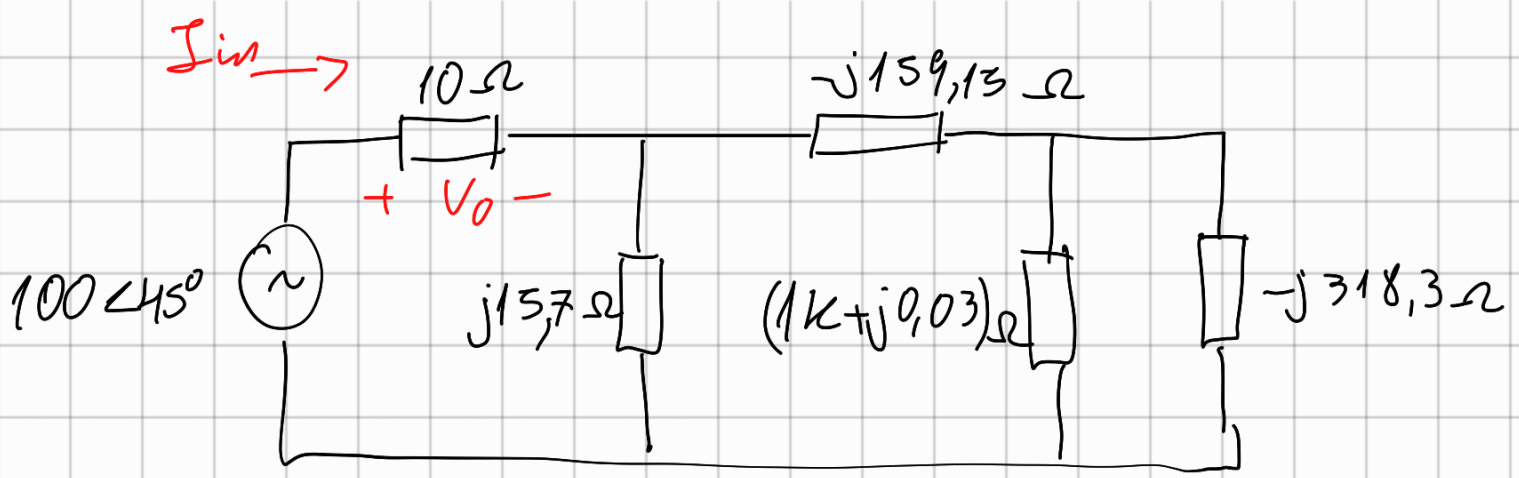


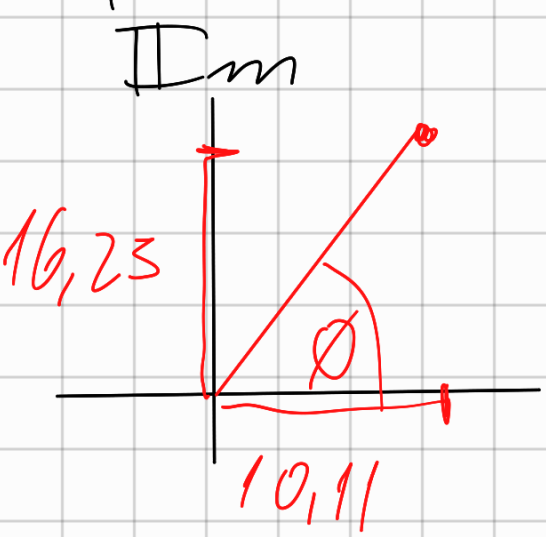
Figura 4: Circuito Problema 4.

Encuentre $V_0(t)$ e $i_0(t)$.



$$Z_{eq} = 10 + \left[j15,7 \,||\, \left(-j159,15 + \left((1K + j0,03) \,||\, -j318,3 \right) \right) \right]$$

$$Z_{eq} = 10,11 + j16,25 \ [\Omega]$$



$$|Z_{eq}| = \sqrt{10,11^2 + 16,25^2}$$

$$= 19,14$$

$$\angle Z_{eq} = \arctan\left(\frac{16,25}{10,11}\right)$$

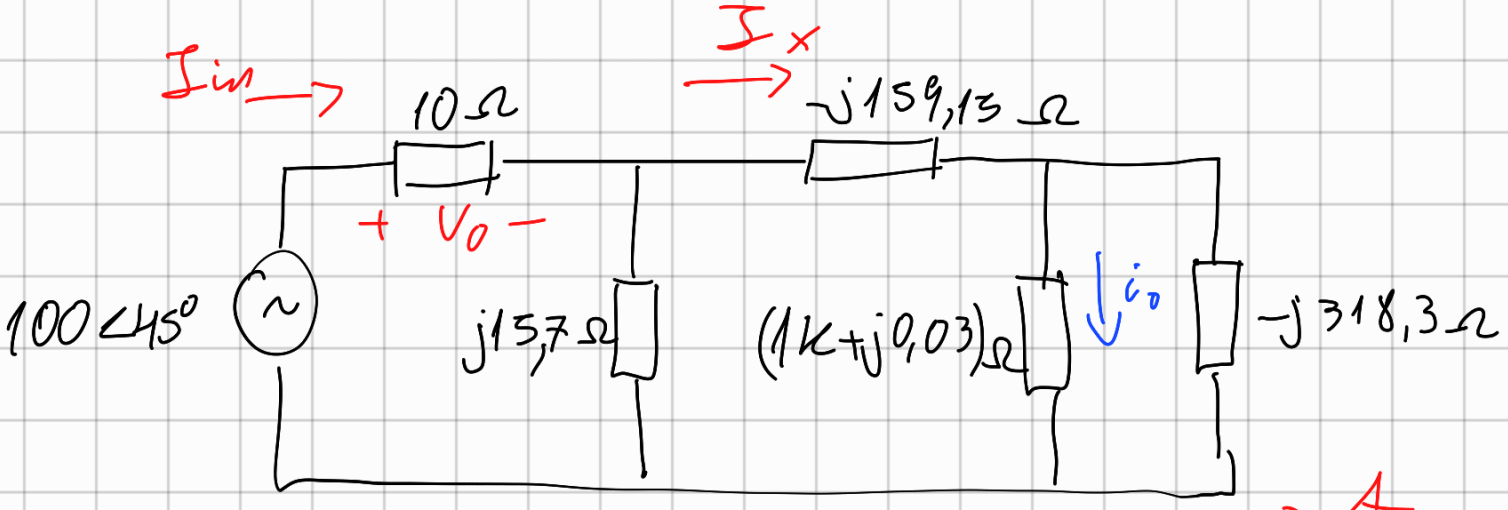
$$= 58,1^\circ$$

$$\hat{Z}_{eq} = 19{,}14 \angle 58{,}1^\circ$$

$$\hat{V}_0 = \frac{\hat{V}_{in}}{\hat{Z}_{eq}} \cdot 10 = \frac{100\angle 45^\circ}{19{,}14\angle 58{,}1^\circ} 10\angle 0^\circ$$

$$\hat{V}_0 = \frac{100 \cdot 10}{19{,}14} \angle 45^\circ - 58{,}1^\circ = 52{,}25 \angle -13{,}1^\circ$$

$$\boxed{V_0(t) = 52{,}25 \cos(100\pi t - 13{,}1^\circ)}$$

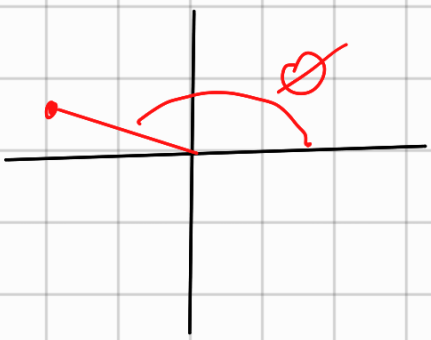


$$I_x = I_{in} \frac{j15{,}7 \rightarrow A}{j15{,}7 + (-j159{,}15 + (1k+j0{,}03) \| -j318{,}3)} \quad (*)$$

$$i_0 = I_x \frac{-j318{,}3}{\underbrace{-j318{,}3 + 1k + j0{,}03}_{B}} \quad (**)$$

Si reemplazamos (*) en (**) y resolvemos:

$$\hat{I}_o = \hat{I}_{in} \underbrace{(-0,00106 + j0,0107)}_{A \cdot B}$$



$$\angle = +180° - \arctan\left(\frac{0,0107}{0,00106}\right)$$

$$= 95,65°$$

$$|AB| = 0,0108$$

$$\hat{I}_o = \hat{I}_{in}\ 0,0108 \angle 95,65°$$

$$\hat{I}_{in} = \frac{\hat{V}_{in}}{\hat{Z}_{eq}} = \frac{100 \angle 45°}{19,14 \angle 58,1°} = 5,22 \angle -13,1°$$

$$\hat{I}_o = 5,22 \angle -13,1° \cdot 0,0108 \angle 95,65°$$

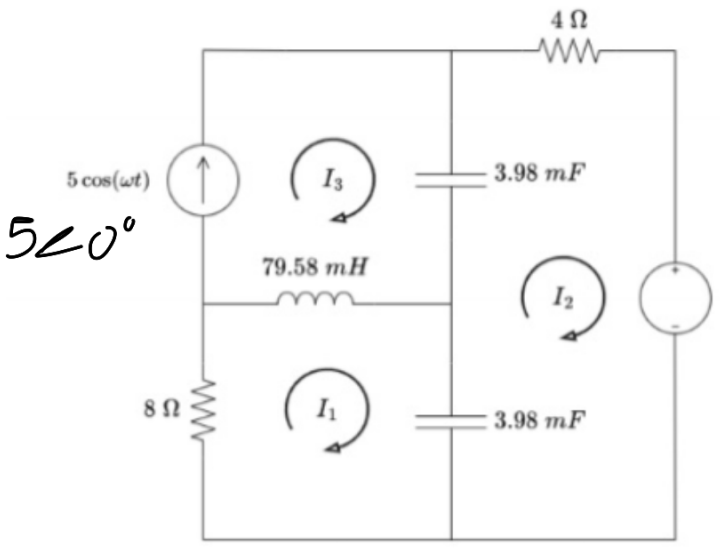
$$\hat{I}_o = 0,056 \angle 82,55° \ [A]$$

$$= 56 \angle 82,55° \ [mA]$$

$$i_o(t) = 56 \cos(100\pi t + 82,55°) \ [mA]$$

Propuesto

Considere el siguiente circuito:



$5\angle 0°$

$\omega = 2\pi \cdot 20 = 125{,}66 \frac{rad}{s}$

$20\angle 90°[V]$

Figura 5: Circuito Problema Propuesto.

Considerando que este opera con frecuencia $f = 20\ Hz$, encuentre I_1, I_2 e I_3 en forma fasorial.

Por mallas:

$$8I_1 + j10(I_1 - I_3) + j2(I_1 - I_2) = 0 \quad (1)$$

$$-j2(I_2 - I_1) + -j2(I_2 - I_3) + 4I_2 + 20\angle 90° = 0 \quad (2)$$

Además $I_3 = 5\angle 0°\ [A]$

$$\Rightarrow 8I_1 + j10(I_1 - 5\angle 0°) + j2(I_1 - I_2) = 0$$

$$-j2(I_2 - I_1) + -j2(I_2 - 5\angle 0°) + 4I_2 + 20\angle 90° = 0$$

Si resolvemos:

$$(8 + j8)I_1 + (j2)I_2 = 50\angle 90°$$

$$(j2)I_1 + (4 - j4)I_2 = -(30\angle 90°)$$

Si se resuelve:

$I_1 = 3,59 \angle 55,01° \ [A] \rightarrow$

$\pi \rightarrow 180°$

$x \rightarrow 55,01°$

$x = 0,96 \ rad$

$I_2 = 6,12 \angle -35,22° \ [A] \rightarrow$

$\pi \rightarrow 180°$

$x \rightarrow -35,22$

$x = -0,61 \ rad$

$\Rightarrow i_1(t) = 3,59 \cos(125,66t + \underbrace{0,96}_{\text{radianes}}) \ [A]$

$\Rightarrow i_2(t) = 6,12 \cos(125,66t - \underbrace{0,61}_{\text{radianes}}) \ [A]$

$\Rightarrow i_3(t) = 5 \cos(125,66t) \ [A]$