IEE 2103 Señales y Sistemas

Clase 4B: Transformada de Laplace: ejemplos, ROC e inversa

4.3 Ejemplos de LT

LT del escalón

$$
\begin{aligned}
F(s) &= \int_{-\infty}^{\infty} \sqcap(t) e^{-st}\, dt \\
&= \int_{0}^{\infty} e^{-st}\, dt \\
&= \left. \frac{e^{-st}}{-s} \right|_{0}^{\infty} \\
&= \frac{1}{s} \quad \mathrm{Re}\{s\} > 0
\end{aligned}
$$

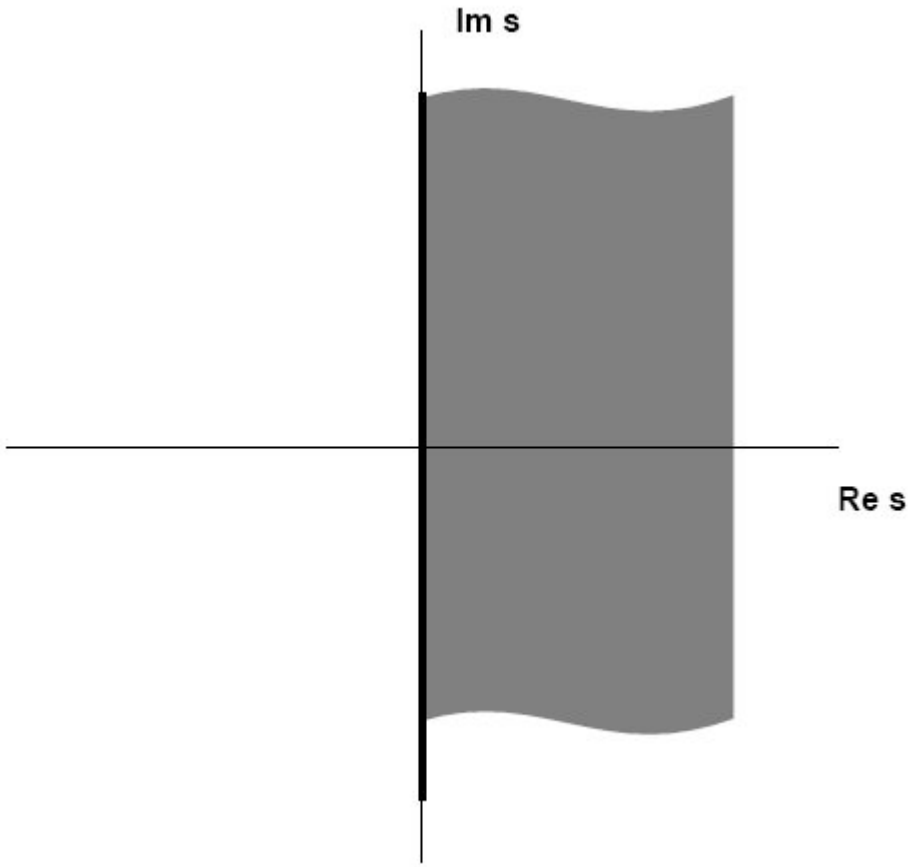
4.3 Ejemplos de LT

LT del escalón

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\begin{aligned}
F(s) &= \int_{-\infty}^{\infty} \sqcap(t) e^{-st}\, dt \\
&= \int_{0}^{\infty} e^{-st}\, dt \\
&= \frac{e^{-st}}{-s}\Big|_{0}^{\infty} \qquad e^{-i2\pi ut} \; e^{-\sigma t}\Big|_{0}^{\infty} \\
&= \frac{1}{s} \qquad \mathrm{Re}\{s\} > 0
\end{aligned}
$$

4.3 Ejemplos de LT

ROC de la LT del escalón



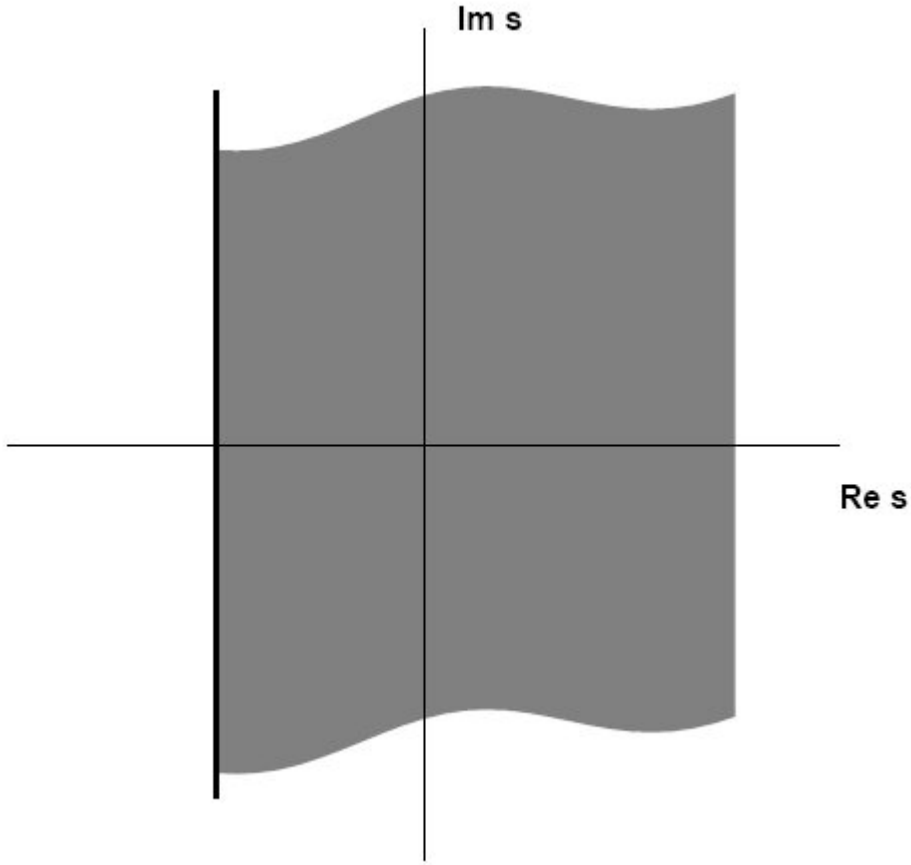
4.3 Ejemplos de LT

LT de la función exponencial

$$
\begin{aligned}
F(s) &= \int_{-\infty}^{\infty} e^{-at} \sqsubset(t) e^{-st}\, dt \\
&= \int_{0}^{\infty} e^{-(a+s)t}\, dt \\
&= \frac{1}{s+a} \quad \mathrm{Re}\{s\} > -a
\end{aligned}
$$

4.3 Ejemplos de LT

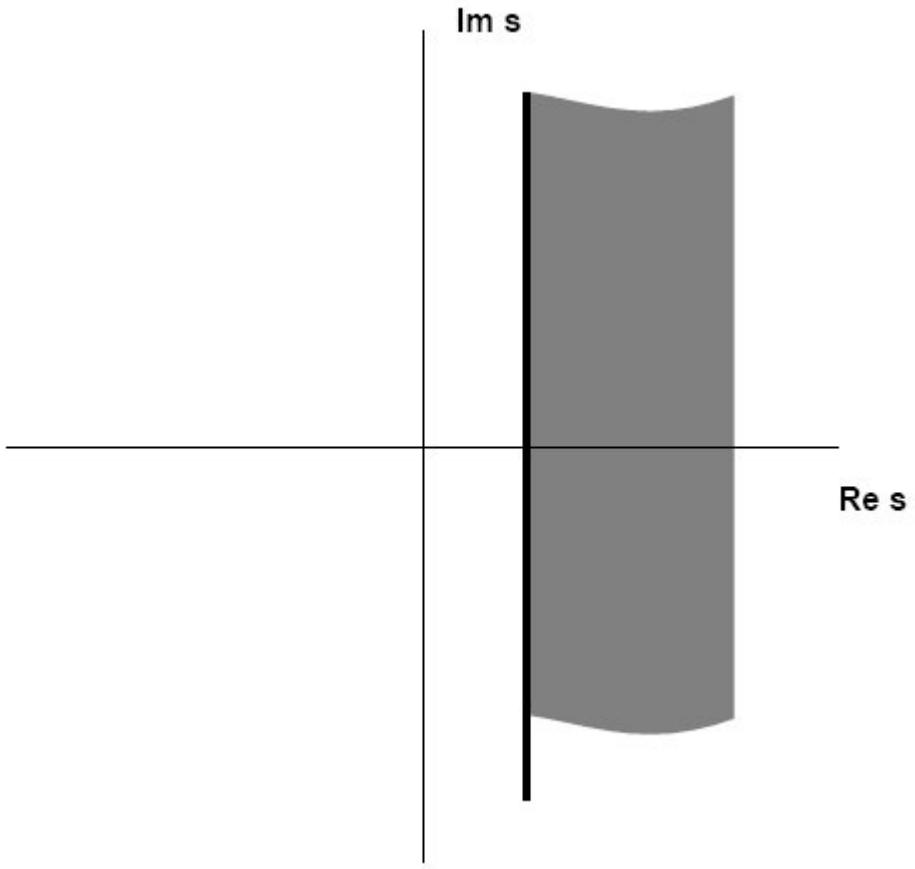
ROC de la LT de la función exponencial



$a > 0$

4.3 Ejemplos de LT

ROC de la LT de la función exponencial



$a < 0$

4.3 Ejemplos de LT

LT de $f(t) = e^{-|t|}$

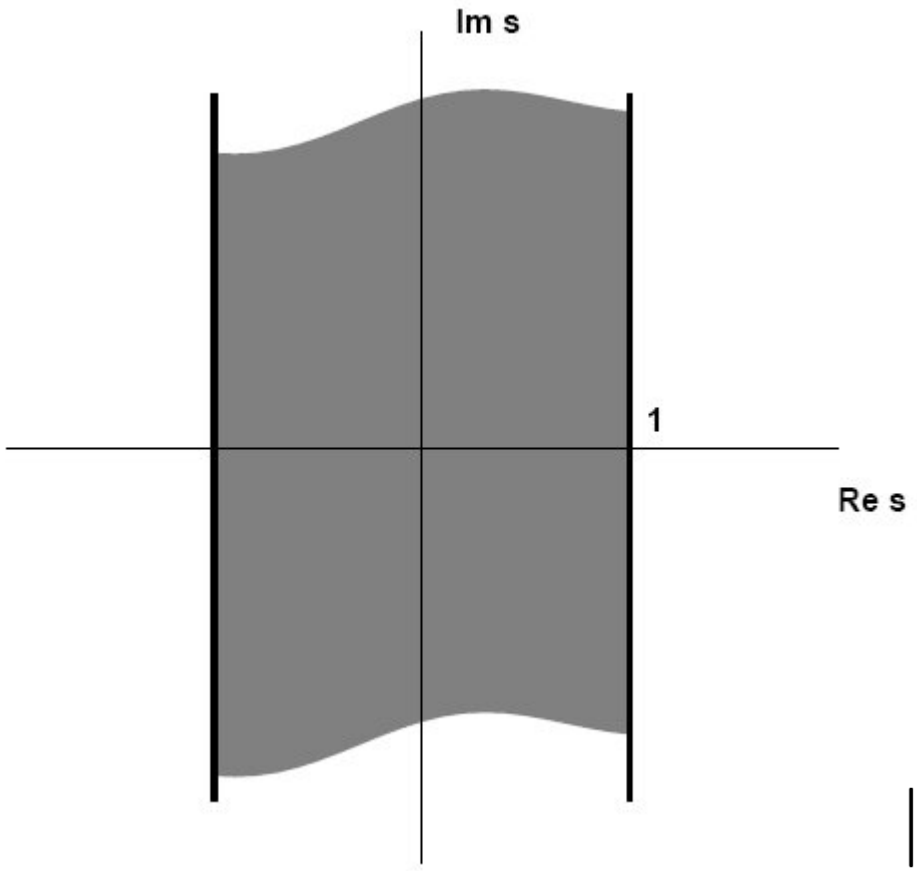
$$
\begin{aligned}
F(s) &= \int_{-\infty}^{\infty} e^{-|t|} e^{-st}\, dt \\
&= \int_{-\infty}^{\infty} e^{-|t|} e^{-\sigma t} e^{-i2\pi u t}\, dt \\
&= \int_{-\infty}^{0} e^{(1-\sigma)t} e^{-i2\pi u t}\, dt \qquad \sigma < 1 \\
&\quad + \int_{0}^{\infty} e^{-(1+\sigma)t} e^{-i2\pi u t}\, dt \qquad \sigma > -1
\end{aligned}
$$

4.3 Ejemplos de LT

$$
\begin{aligned}
F(s) &= \int_{-\infty}^{0} e^{(1-\sigma)t} e^{-i2\pi ut}\, dt && \sigma < 1 \\
&\quad + \int_{0}^{\infty} e^{-(1+\sigma)t} e^{-i2\pi ut}\, dt && \sigma > -1 \\
&= \frac{1}{1-s} + \frac{1}{1+s} \\
&= \frac{2}{1-s^2} && |\sigma| < 1
\end{aligned}
$$

4.3 Ejemplos de LT

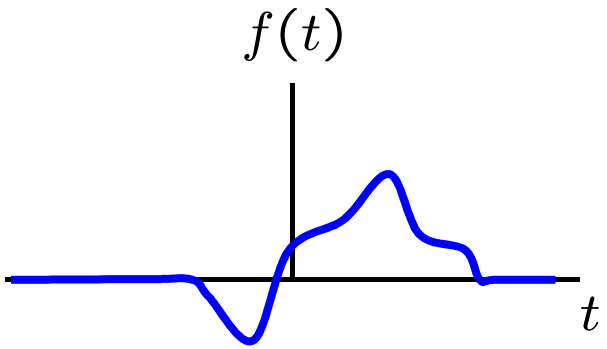
ROC

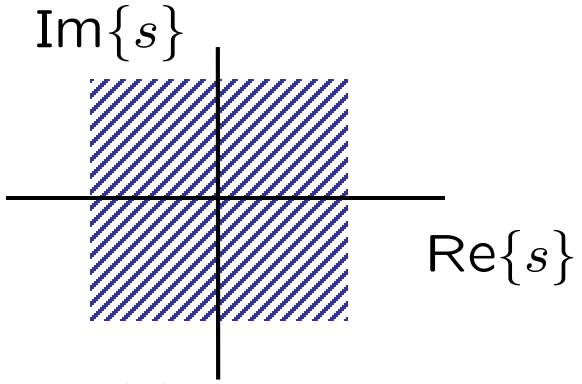


$$|\sigma| < 1$$

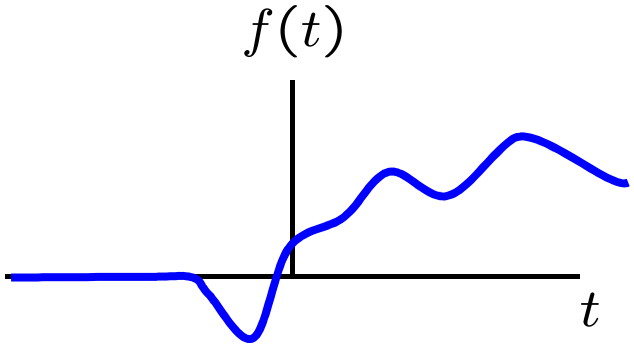
4.4 Región de Convergencia

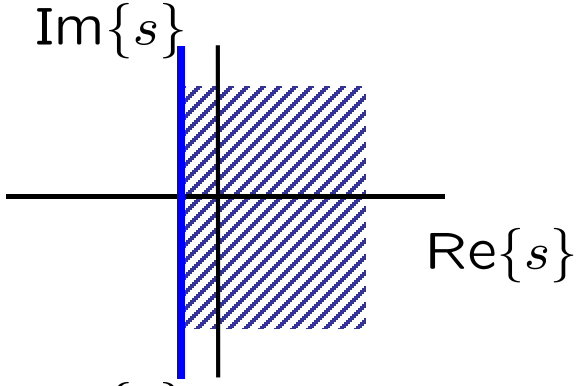
$f(t)$ integrable



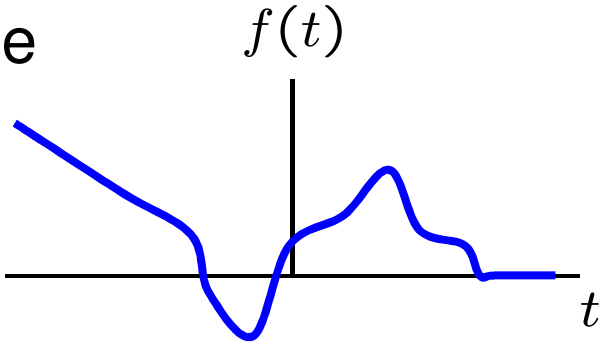


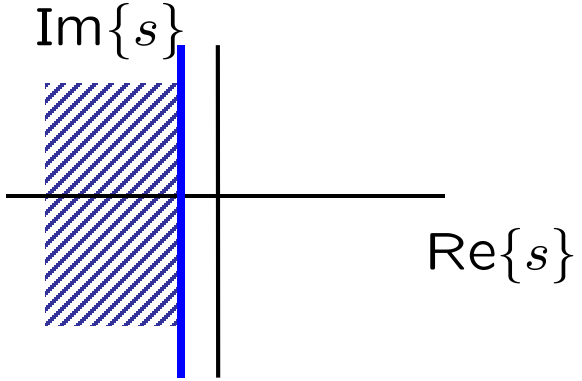
$f(t)$ integrable





$f(t)$ no integrable





4.5 Polos y ceros

Polos: valor de *s* para el cual *F(s)* diverge

- En el límite de la ROC hay un polo
- No hay polos en la ROC

Ceros: valor de *s* para el cual *F(s)=0*

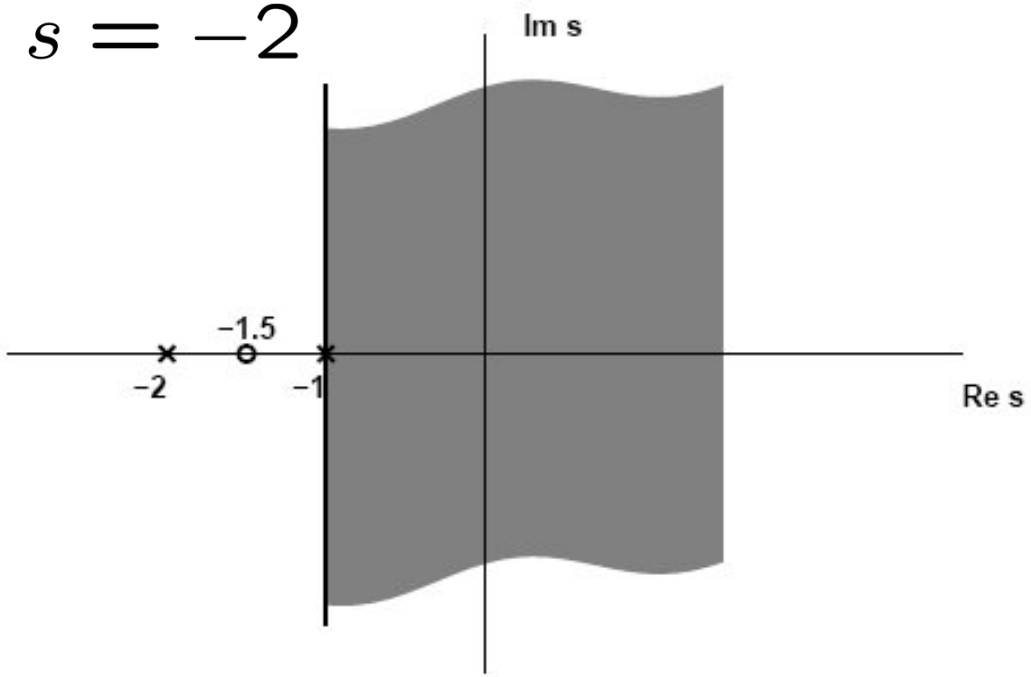
4.5 Polos y ceros

Ejemplo $f(t) = (e^{-t} + e^{-2t})\sqcap(t)$

$$\longrightarrow \frac{2s+3}{s^2+3s+2} \qquad \sigma > -1$$

Polos en $s = -1$ y $s = -2$

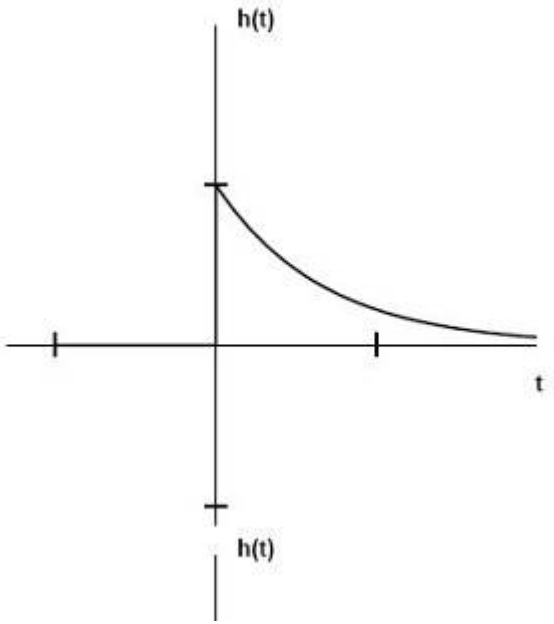
Cero en $s = -3/2$

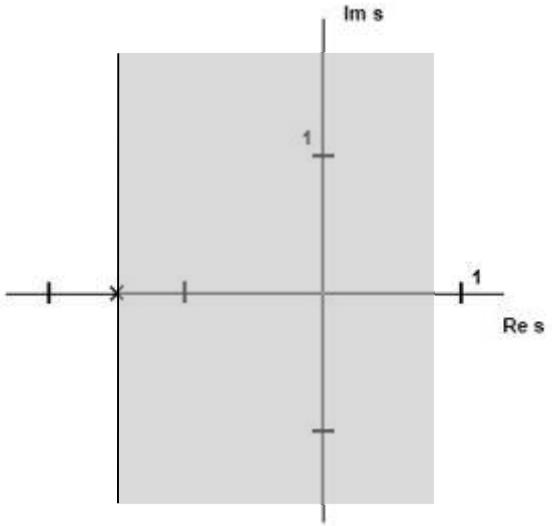


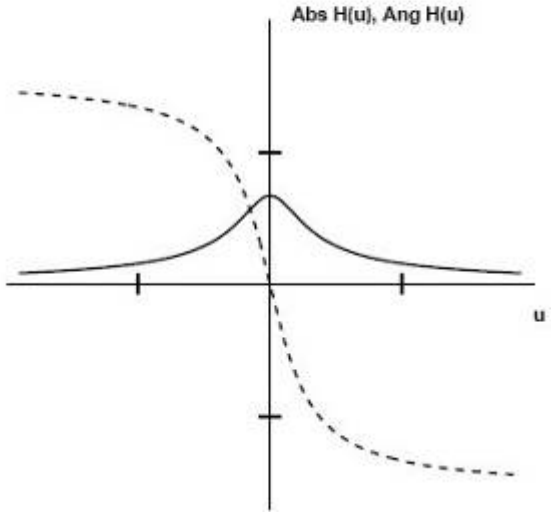
4.5 Polos y ceros

Efecto de un polo

$$h(t) = e^{pt} \sqcap (t) \longrightarrow H(s) = \frac{1}{s - p} \qquad \sigma > p$$







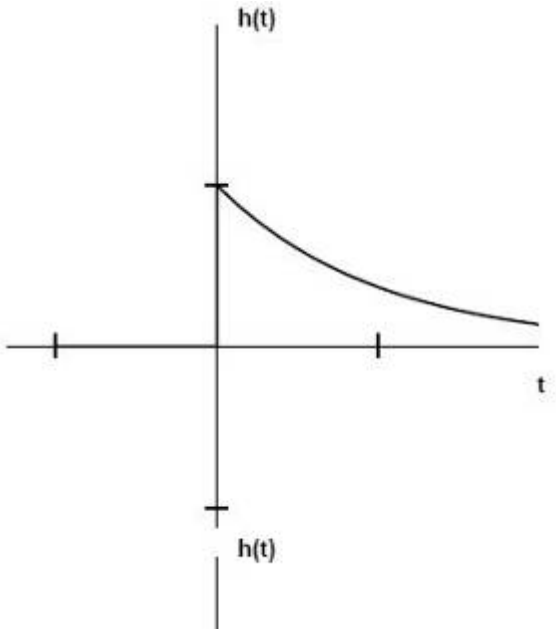
$$p = -1.5$$

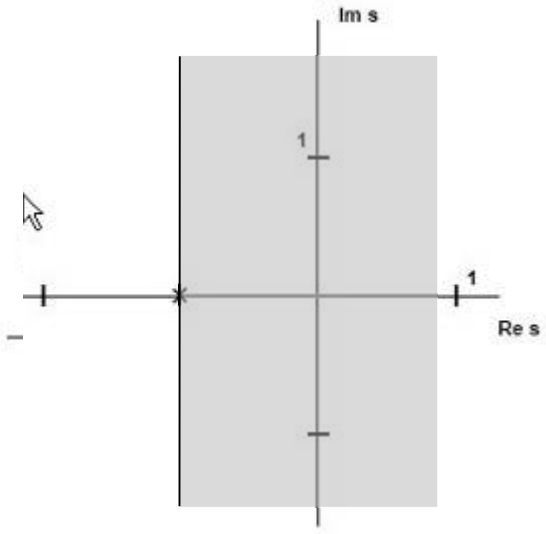
Fourier

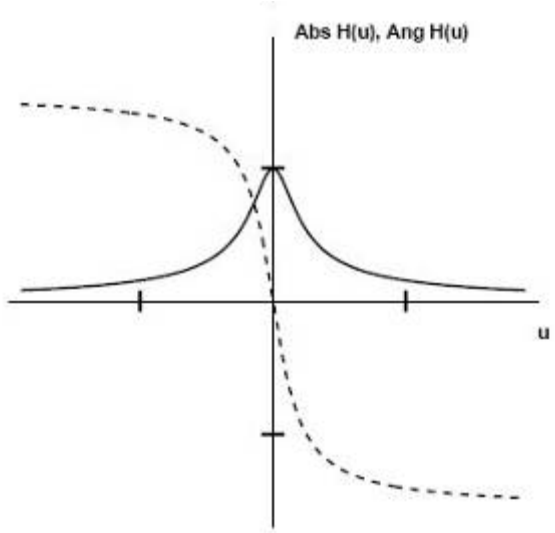
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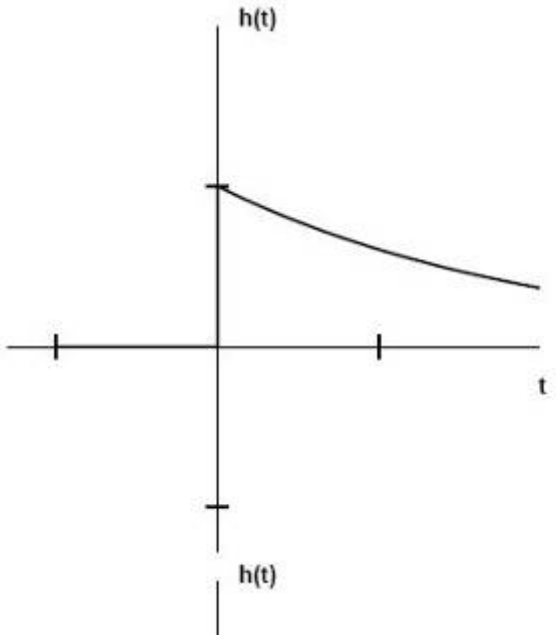
$$p = -1$$

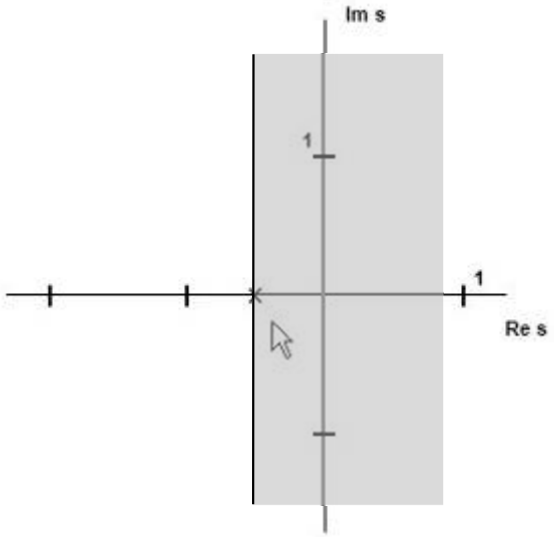
Fourier

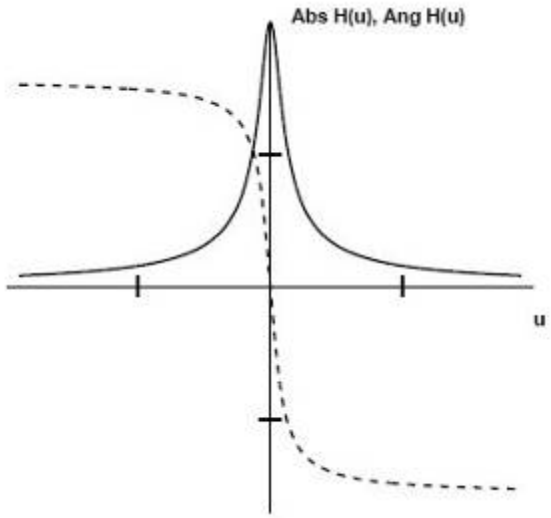
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Efecto de un polo

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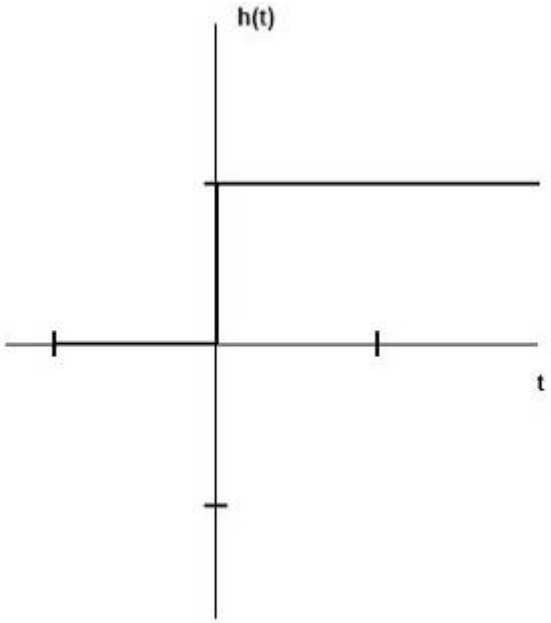
$$p = -0.5$$

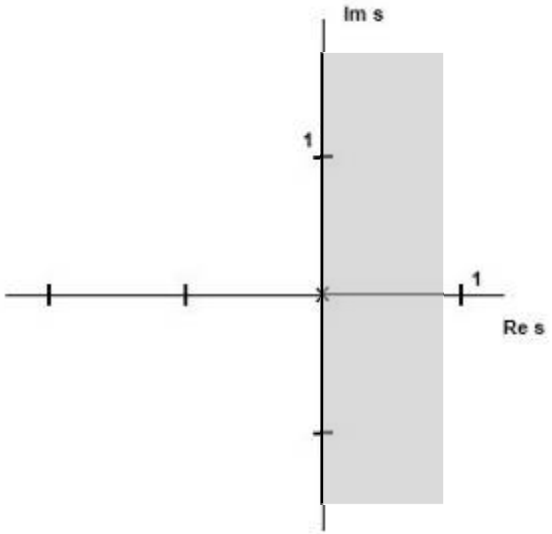
Fourier

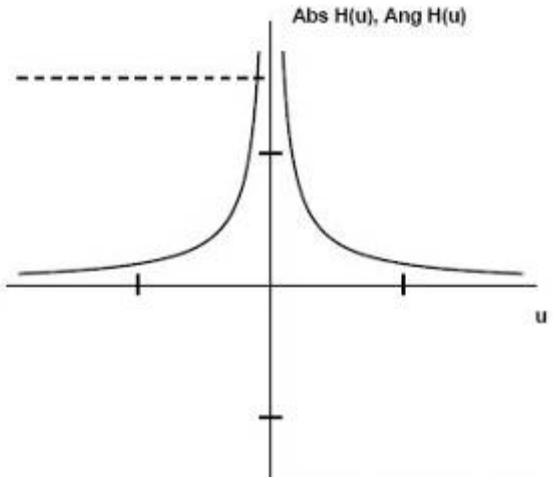
4.5 Polos y ceros

Efecto de un polo

$$h(t) = e^{pt} \sqcap(t) \longrightarrow H(s) = \frac{1}{s-p} \qquad \sigma > p$$







$$p = 0$$

Fourier

4.6 LT inversa

$$F(s) = \int_{-\infty}^{\infty} f(t)e^{-st}\,dt \qquad \alpha < Re\{s\} < \beta$$

$$f(t) = \frac{1}{i2\pi}\int_{c-i\infty}^{c+i\infty} F(s)e^{st}\,ds \qquad \alpha < c < \beta$$

4.6 LT inversa

$$
\begin{aligned}
\mathcal{F}\{f(t)e^{-\sigma t}\} &= F(s) \quad /\mathcal{F}^{-1} \\
f(t)e^{-\sigma t} &= \mathcal{F}^{-1}\{F(s)\} \\
f(t) &= e^{\sigma t}\mathcal{F}^{-1}\{F(s)\} \\
&= e^{\sigma t}\int_{-\infty}^{\infty} F(\sigma + i2\pi u)e^{i2\pi u t}\,du \\
&= \int_{-\infty}^{\infty} F(\sigma + i2\pi u)e^{(\sigma+i2\pi u)t}\,du
\end{aligned}
$$

y sustituyendo $s = \sigma + i2\pi u \qquad du = \dfrac{ds}{i2\pi}$

4.6 LT inversa

$$f(t) = \frac{1}{i2\pi} \int_{c-i\infty}^{c+i\infty} F(s) e^{st}\, ds$$

$$= \int_{-\infty}^{\infty} F(\sigma + i2\pi u) e^{(\sigma + i2\pi u)t}\, du$$

y sustituyendo $s = \sigma + i2\pi u \qquad du = \frac{ds}{i2\pi}$

4.6 LT inversa

La transformada de Laplace inversa puede obtenerse por:

- aplicación directa de la fórmula inversa
- aplicación de propiedades para reducir a un par conocido
- fracciones parciales, si es que tiene forma de fracción polinomial