IEE 2103 Señales y Sistemas

Clase 6C: Transformada de Fourier Discreta (matricial)

6.4 Transformada de Fourier Discreta

Interpretación matricial

$$\mathbf{F} = \mathcal{W}\mathbf{f}$$

$$\mathbf{F} = \begin{bmatrix} F[0] \\ F[1] \\ F[2] \\ \vdots \\ F[N-1] \end{bmatrix} \qquad \mathbf{f} = \begin{bmatrix} f[0] \\ f[1] \\ f[2] \\ \vdots \\ f[N-1] \end{bmatrix}$$

6.4 Transformada de Fourier Discreta

Interpretación matricial

$$\mathbf{F} = \mathcal{W}\mathbf{f}$$

$$\mathcal{W} = \frac{1}{N}\begin{bmatrix} W^{0\cdot 0} & W^{1\cdot 0} & W^{2\cdot 0} & \dots & W^{(N-1)\cdot 0} \\ W^{0\cdot 1} & W^{1\cdot 1} & W^{2\cdot 1} & \dots & W^{(N-1)\cdot 1} \\ W^{0\cdot 2} & W^{1\cdot 2} & W^{2\cdot 2} & \dots & W^{(N-1)\cdot 2} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ W^{0\cdot (N-1)} & W^{1\cdot (N-1)} & W^{2\cdot (N-1)} & \dots & W^{(N-1)\cdot (N-1)} \end{bmatrix}$$

$$W = e^{-i2\pi/N} \qquad W^N = W^0 = 1$$

$$|W^k| = 1 \qquad W^{k+N} = W^k$$

6.4 Transformada de Fourier Discreta

Ejemplo para N=4

$$W = e^{-i\pi/2}$$

$$\mathcal{W} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-i2\pi\frac{1}{4}} & e^{-i2\pi\frac{1}{2}} & e^{-i2\pi\frac{3}{4}} \\ 1 & e^{-i2\pi\frac{1}{2}} & e^{-i2\pi} & e^{-i2\pi\frac{3}{2}} \\ 1 & e^{-i2\pi\frac{3}{4}} & e^{-i2\pi\frac{3}{2}} & e^{-i2\pi\frac{9}{4}} \end{bmatrix}$$

$$W = e^{-i\pi/2} = -i$$

6.4 Transformada de Fourier Discreta

Ejemplo para N=4

$$W = e^{-i\pi/2} = -i$$

$$\mathcal{W} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -i \end{bmatrix}$$

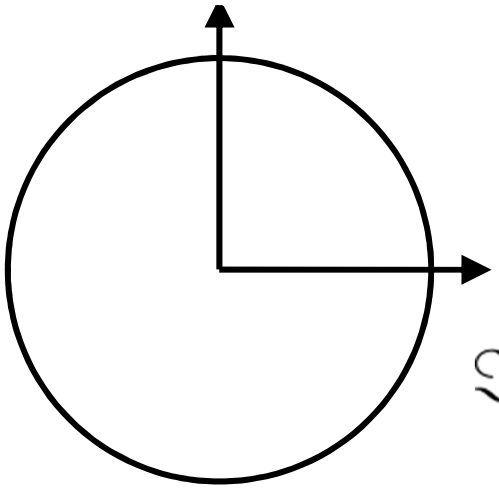
6.4 Transformada de Fourier Discreta

Ejemplo para N=4

$$W^{kn} = \cos(2\pi\frac{kn}{N}) - i\,\text{sen}\,(2\pi\frac{kn}{N})$$

$$\mathcal{W} = \frac{1}{4}\begin{bmatrix} \uparrow & \uparrow & \uparrow & \uparrow \\ \uparrow & \leftarrow & \downarrow & \rightarrow \\ \uparrow & \downarrow & \uparrow & \downarrow \\ \uparrow & \rightarrow & \downarrow & \leftarrow \end{bmatrix}$$

$$\Re\{W^{kn}\} = \cos(2\pi\frac{kn}{N})$$



$$\Im\{W^{kn}\} = -\,\text{sen}\,(2\pi\frac{kn}{N})$$