IEE 2103 Señales y Sistemas

Clase 6B: Transformada de Fourier Discreta

6.4 Transformada de Fourier Discreta

La DFT es equivalente a la

- DTFT si se discretiza la frecuencia,
- o a la DFFT si se discretiza el tiempo.

6.4 Transformada de Fourier Discreta

Discretizando el tiempo en la DFFT

$$\tilde{f}(x) \longrightarrow F[k]$$

$$\tilde{f}[n] = T\tilde{f}(nT)$$

$$\tilde{f}_s(x) = \sqcup\!\sqcup(x/T)\tilde{f}(x) = T\sum_{n=-\infty}^{\infty} \tilde{f}(x)\delta(x - nT)$$

6.4 Transformada de Fourier Discreta

La DFFT de $\tilde{f}_s(x)$ es

$$\begin{aligned}\tilde{F}[k] &= U\int_0^{1/U}\tilde{f}_s(x)e^{-i2\pi kUx}\,dx \\ &= UT\int_0^{1/U}\sum_{n=-\infty}^{\infty}\tilde{f}(nT)\delta(x-nT)e^{-i2\pi kUx}\,dx\end{aligned}$$

$$\tilde{F}[k] = UT\int_{-\infty}^{\infty}\sum_{n=0}^{N-1}\tilde{f}(nT)\delta(x-nT)e^{-i2\pi kUx}\,dx$$

6.4 Transformada de Fourier Discreta

La DFFT de $\tilde{f}_s(x)$ es

¿Por qué N? Lo veremos enseguida.

$$\tilde{F}[k] = UT \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} \tilde{f}(nT)\delta(x - nT)e^{-i2\pi kUx}\, dx$$

6.4 Transformada de Fourier Discreta

$$
\begin{aligned}
\tilde{F}[k] &= UT \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} \tilde{f}(nT)\delta(x-nT)e^{-i2\pi kUx}\,dx \\
&= UT \sum_{n=0}^{N-1} \int_{-\infty}^{\infty} \tilde{f}(nT)\delta(x-nT)e^{-i2\pi kUx}\,dx \\
&= UT \sum_{n=0}^{N-1} \tilde{f}(nT) \int_{-\infty}^{\infty} \delta(x-nT)e^{-i2\pi kUx}\,dx \\
&= UT \sum_{n=0}^{N-1} \tilde{f}(nT)e^{-i2\pi kUnT}
\end{aligned}
$$

6.4 Transformada de Fourier Discreta

$$\tilde{f}(nT) = \frac{1}{T}\tilde{f}[n]$$

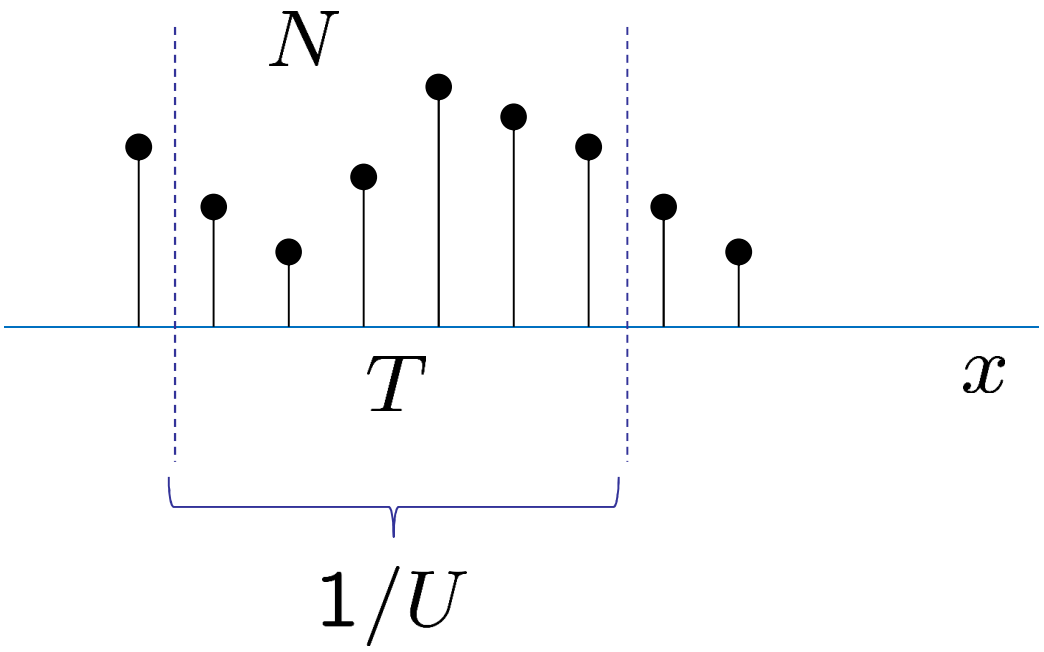
$$\tilde{F}[k] = \frac{1}{NT}\sum_{n=0}^{N-1}\tilde{f}[n]e^{-i2\pi kn/N}$$

DFT

$$\tilde{F}[k] = \frac{1}{N}\sum_{n=0}^{N-1}\tilde{f}[n]e^{-i2\pi kn/N}$$

6.4 Transformada de Fourier Discreta

$$\tilde{F}[k] = UT \int_{-\infty}^{\infty} \sum_{n=0}^{N-1} \tilde{f}(nT)\delta(x - nT)e^{-i2\pi kUx}\, dx$$



El periodo debe ser un múltiplo entero de T

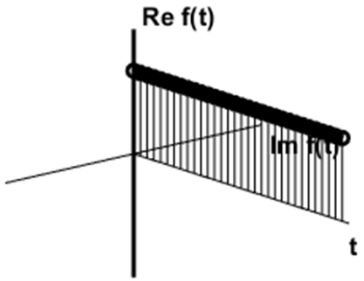
$$1/U = NT$$

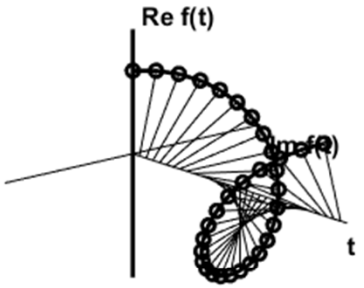
$$NUT = 1$$

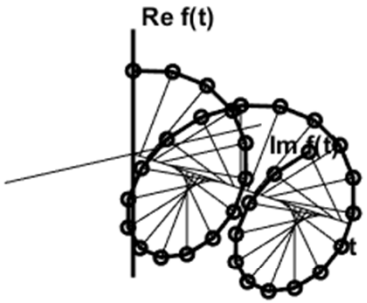
6.4 Transformada de Fourier Discreta

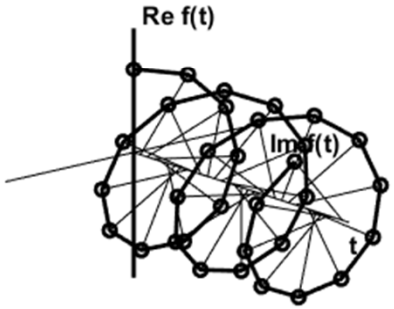
bases de la DFT

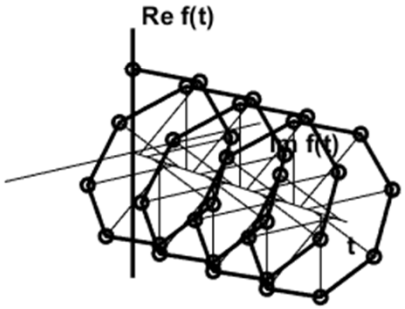
$$W_k[n] = e^{-i2\pi kn/N}$$

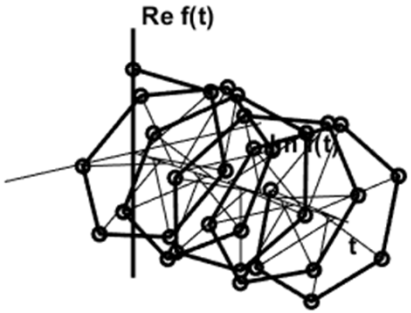












6.4 Transformada de Fourier Discreta

Discretizando la frecuencia en la DTFT

$$f[n] \longrightarrow \tilde{F}(u)$$

$$\tilde{F}[k] = U\,\tilde{F}(kU)$$

$$\tilde{F}_s(u) = U \sum_{k=-\infty}^{\infty} \tilde{F}(u)\delta(u - kU).$$

6.4 Transformada de Fourier Discreta

La DTFT de $\tilde{F}_s(u)$ es

$$
\begin{aligned}
\tilde{f}[n] &= T \int_0^{1/T} \tilde{F}_s(u) e^{i2\pi unT}\, du \\
&= UT \int_0^{1/T} \sum_{n=-\infty}^{\infty} \tilde{F}(u)\delta(u - kU) e^{i2\pi unT}\, du \\
&= UT \sum_{k=0}^{N-1} \int_{-\infty}^{\infty} \tilde{F}(kU)\delta(u - kU) e^{i2\pi unT}\, du \\
&= UT \sum_{k=0}^{N-1} \tilde{F}(kU) e^{i2\pi kUnT}
\end{aligned}
$$

6.4 Transformada de Fourier Discreta

$$\tilde{f}[n] = T \sum_{k=0}^{N-1} \tilde{F}[k] e^{i2\pi kn/N}$$

DFT inversa

$$\tilde{f}[n] = \sum_{k=0}^{N-1} \tilde{F}[k] e^{i2\pi kn/N}$$

Resumen por dominios

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi ux}\, dx$$

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{i2\pi ux}\, du$$

FT – ***x*** : continuo ***u*** : continuo

$$F[k] = U \int_{0}^{1/U} \tilde{f}(x) e^{-i2\pi kUx}\, dx$$

$$\tilde{f}(x) = \sum_{k=-\infty}^{\infty} F[k] e^{i2\pi kUx}$$

DFFT – ***x*** : continuo ***u*** : discreto (***k***)

$$\tilde{F}(u) = \sum_{n=-\infty}^{\infty} f[n] e^{-i2\pi unT}$$

$$f[n] = T \int_{0}^{1/T} \tilde{F}(u) e^{i2\pi unT}\, du.$$

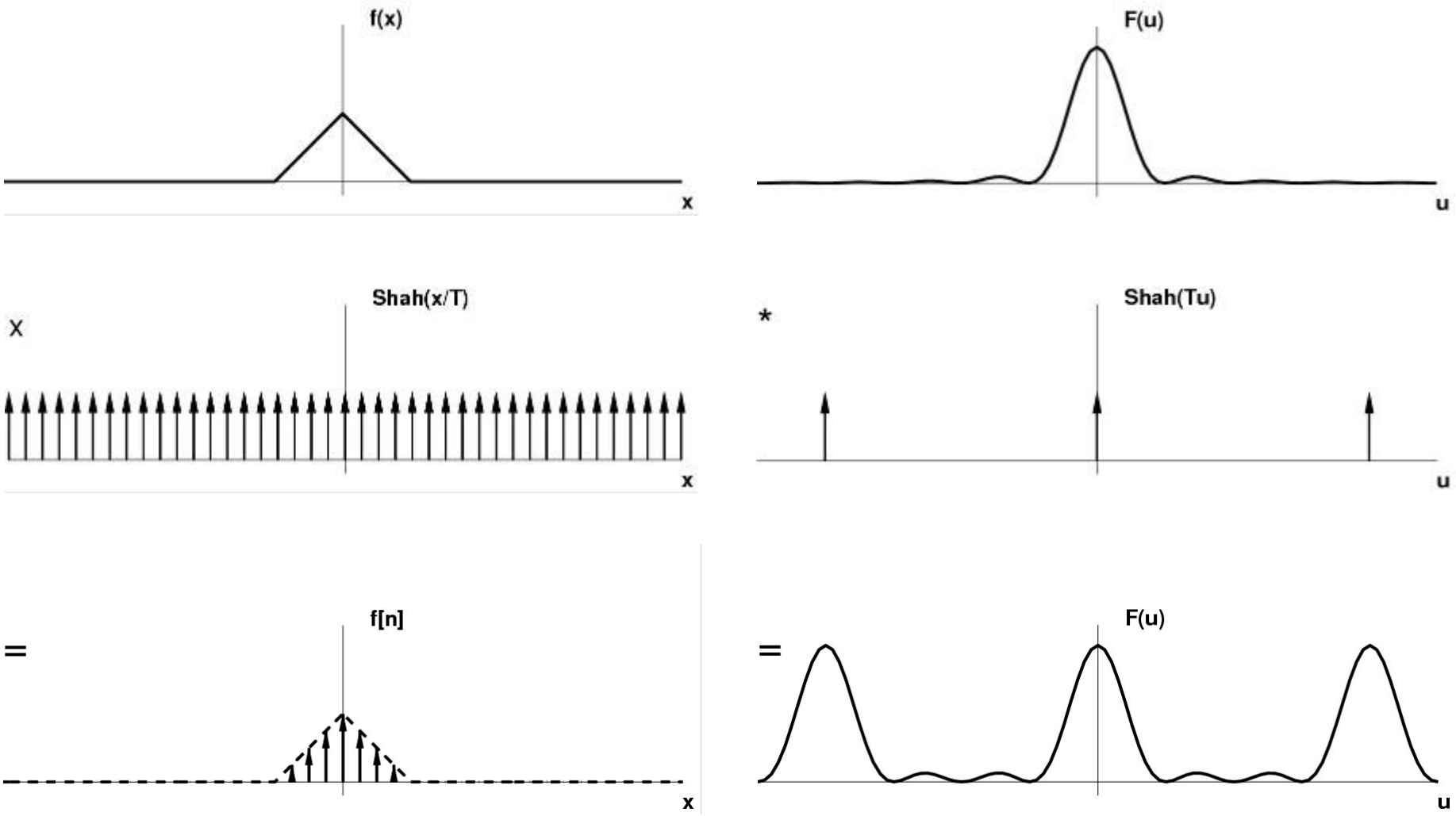
DTFT – ***x*** : discreto (***n***) ***u*** : continuo

$$\tilde{F}[k] = \frac{1}{N} \sum_{n=0}^{N-1} \tilde{f}[n] e^{-i2\pi kn/N}$$

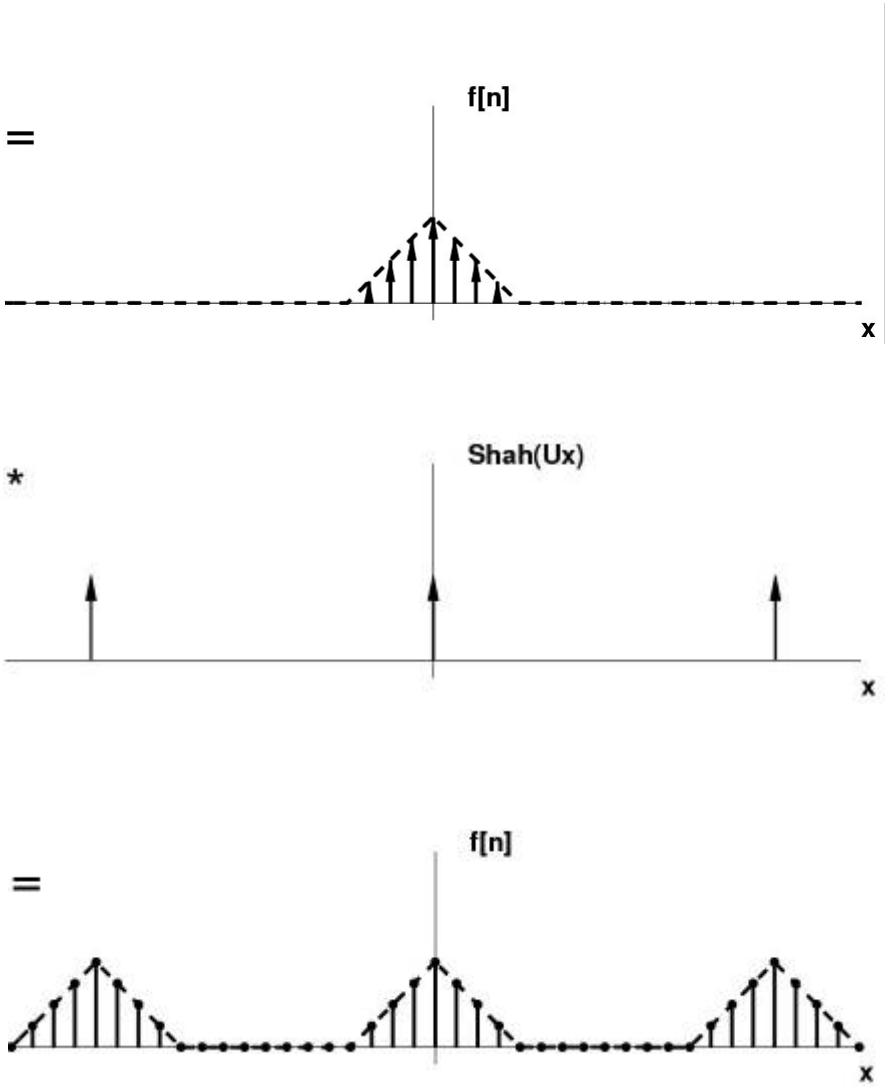
$$\tilde{f}[n] = \sum_{k=0}^{N-1} \tilde{F}[k] e^{i2\pi kn/N}$$

DFT – ***x*** : discreto (***n***) ***u*** : discreto (***k***)

6.4 Transformada de Fourier Discreta



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6.4 Transformada de Fourier Discreta

$$\tilde{F}[k] = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{f}[n] e^{-i2\pi kn/N} \qquad f[n] = (1,\ 1,\ 0,\ 0)$$

$$\begin{aligned} & F[0] \\ &= \frac{1}{4}\left\{1 \cdot e^{-i2\pi \cdot 0 \cdot 0/4} + 1 \cdot e^{-i2\pi \cdot 0 \cdot 1/4} + 0 \cdot e^{-i2\pi \cdot 0 \cdot 2/4} + 0 \cdot e^{-i2\pi \cdot 0 \cdot 3/4}\right\} \\ &= \frac{1}{4}\{1 \cdot 1 + 1 \cdot 1 + 0 \cdot 1 + 0 \cdot 1\} = \frac{1}{4}2 \end{aligned}$$

6.4 Transformada de Fourier Discreta

$$\tilde{F}[k] = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{f}[n] e^{-i2\pi kn/N} \qquad f[n] = (1,\ 1,\ 0,\ 0)$$

$$\begin{aligned} & F[1] \\ &= \frac{1}{4}\left\{1 \cdot e^{-i2\pi \cdot 1 \cdot 0/4} + 1 \cdot e^{-i2\pi \cdot 1 \cdot 1/4} + 0 \cdot e^{-i2\pi \cdot 1 \cdot 2/4} + 0 \cdot e^{-i2\pi \cdot 1 \cdot 3/4}\right\} \\ &= \frac{1}{4}\{1 \cdot 1 + 1 \cdot (-i) + 0 \cdot (-1) + 0 \cdot i\} = \frac{1}{4}(1 - i) \end{aligned}$$

6.4 Transformada de Fourier Discreta

$$\tilde{F}[k] = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{f}[n] e^{-i2\pi kn/N} \qquad f[n] = (1,\ 1,\ 0,\ 0)$$

$$\begin{aligned} & F[2] \\ &= \frac{1}{4}\left\{1 \cdot e^{-i2\pi \cdot 2 \cdot 0/4} + 1 \cdot e^{-i2\pi \cdot 2 \cdot 1/4} + 0 \cdot e^{-i2\pi \cdot 2 \cdot 2/4} + 0 \cdot e^{-i2\pi \cdot 2 \cdot 3/4}\right\} \\ &= \frac{1}{4}\{1 \cdot 1 + 1 \cdot (-1) + 0 \cdot 1 + 0 \cdot (-1)\} = \frac{1}{4} 0 \end{aligned}$$

6.4 Transformada de Fourier Discreta

$$\tilde{F}[k] = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{f}[n] e^{-i2\pi kn/N} \qquad f[n] = (1,\ 1,\ 0,\ 0)$$

$$F[3]$$
$$= \frac{1}{4}\left\{1 \cdot e^{-i2\pi \cdot 3 \cdot 0/4} + 1 \cdot e^{-i2\pi \cdot 3 \cdot 1/4} + 0 \cdot e^{-i2\pi \cdot 3 \cdot 2/4} + 0 \cdot e^{-i2\pi \cdot 3 \cdot 3/4}\right\}$$
$$= \frac{1}{4}\{1 \cdot 1 + 1 \cdot i + 0 \cdot (-1) + 0 \cdot (-i)\} = \frac{1}{4}(1+i)$$

6.4 Transformada de Fourier Discreta

$$\tilde{F}[k] = \frac{1}{N}\sum_{n=0}^{N-1} \tilde{f}[n] e^{-i2\pi kn/N} \qquad f[n] = (1,\ 1,\ 0,\ 0)$$

$$\begin{aligned}
F[0] &= \frac{1}{4}\{1\cdot 1 + 1\cdot 1 + 0\cdot 1 + 0\cdot 1\} = \frac{1}{4}2 \\
F[1] &= \frac{1}{4}\{1\cdot 1 + 1\cdot (-i) + 0\cdot (-1) + 0\cdot i\} = \frac{1}{4}(1-i) \\
F[2] &= \frac{1}{4}\{1\cdot 1 + 1\cdot (-1) + 0\cdot 1 + 0\cdot (-1)\} = \frac{1}{4}0 \\
F[3] &= \frac{1}{4}\{1\cdot 1 + 1\cdot i + 0\cdot (-1) + 0\cdot (-i)\} = \frac{1}{4}(1+i)
\end{aligned}$$