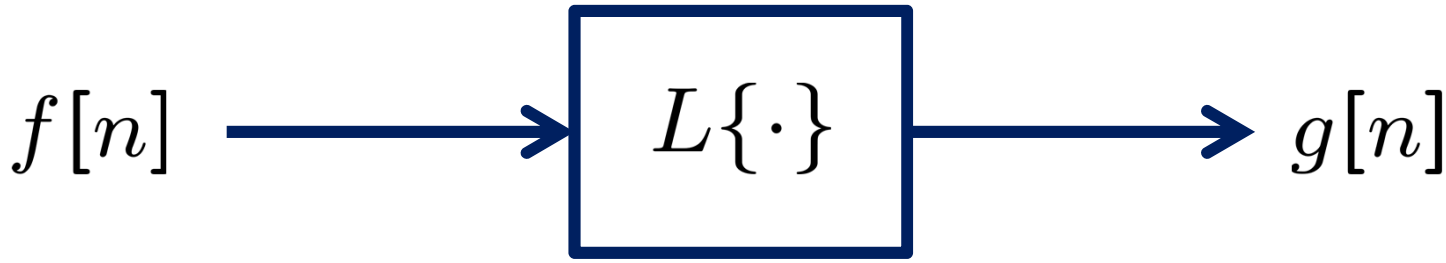
IEE 2103 Señales y Sistemas

Clase 5B: Variables de estado

5.7 Representación de sistemas

Sistemas lineales e invariantes: ecuaciones de diferencias



$$g[n] + \sum_{j=1}^{N} a_j g[n-j] = \sum_{j=0}^{M} b_j f[n-j]$$

Representación de sistemas

$$g[n] + \sum_{j=1}^{N} a_j g[n-j] = \sum_{j=0}^{M} b_j f[n-j]$$

$$g[0],\ g[-1],\ \ldots\ g[-N+1]$$

$$(1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}) g[n] =$$
$$(b_o + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_M z^{-M}) f[n]$$

Variables de estado

$$g[n] = -a_1 g[n-1] - a_2 g[n-2] + b_o f[n]$$

Sea $v_1[n] = g[n-2]$ y $v_2[n] = g[n-1]$

$$v_2[n+1] = -a_1 v_2[n] - a_2 v_1[n] + b_o f[n]$$

$$\begin{bmatrix} v_1[n+1] \\ v_2[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} v_1[n] \\ v_2[n] \end{bmatrix} + \begin{bmatrix} 0 \\ b_o \end{bmatrix} f[n]$$

$$g[n] = \begin{bmatrix} -a_2 & -a_1 \end{bmatrix} \begin{bmatrix} v_1[n] \\ v_2[n] \end{bmatrix} + b_o f[n]$$

Variables de estado

$$\begin{aligned} \mathbf{v}[n+1] &= \mathbf{A}\mathbf{v}[n] + \mathbf{b}f[n] \\ g[n] &= \mathbf{c}\mathbf{v}[n] + df[n] \end{aligned}$$

$$\begin{bmatrix} \cdot \\ \cdot \end{bmatrix} = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \end{bmatrix} + \begin{bmatrix} \cdot \\ \cdot \end{bmatrix} \cdot$$

$$\cdot = \begin{bmatrix} \cdot & \cdot \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \end{bmatrix} + \cdot\cdot$$

Solución de ecuaciones de estado

$$\mathbf{v}[n+1] = \mathbf{A}\mathbf{v}[n] + \mathbf{b}f[n] \quad \mathbf{v}[0] = \mathbf{v}_o$$

Solución para n = 0

$$\mathbf{v}[1] = \mathbf{A}\mathbf{v}_o + \mathbf{b}f[0]$$

Solución para n = 1

$$\begin{aligned} \mathbf{v}[2] &= \mathbf{A}\mathbf{v}[1] + \mathbf{b}f[1] \\ &= \mathbf{A}\{\mathbf{A}\mathbf{v}_o + \mathbf{b}f[0]\} + \mathbf{b}f[1] \\ &= \mathbf{A}^2\mathbf{v}_o + \mathbf{A}\mathbf{b}f[0] + \mathbf{b}f[1] \end{aligned}$$

Solución para n

$$\mathbf{v}[n] = \mathbf{A}^n\mathbf{v}_o + \mathbf{A}^{n-1}\mathbf{b}f[0] + \cdots + \mathbf{b}f[n-1]$$

Solución de ecuaciones de estado

$$\mathbf{v}[n] = \mathbf{A}^n \mathbf{v}_o + \mathbf{A}^{n-1}\mathbf{b}f[0] + \cdots + \mathbf{b}f[n-1]$$

Sea la matriz de transición de estados $\mathbf{\Phi}[n] = \mathbf{A}^n$

$$\begin{aligned} \mathbf{v}[n] &= \mathbf{\Phi}[n]\mathbf{v}_o + \sum_{j=0}^{n-1} \mathbf{\Phi}[n-1-j]\mathbf{b}f[j] \\ &= \mathbf{\Phi}[n]\mathbf{v}_o + \mathbf{\Phi}[n-1] * \mathbf{b}f[n] \end{aligned}$$

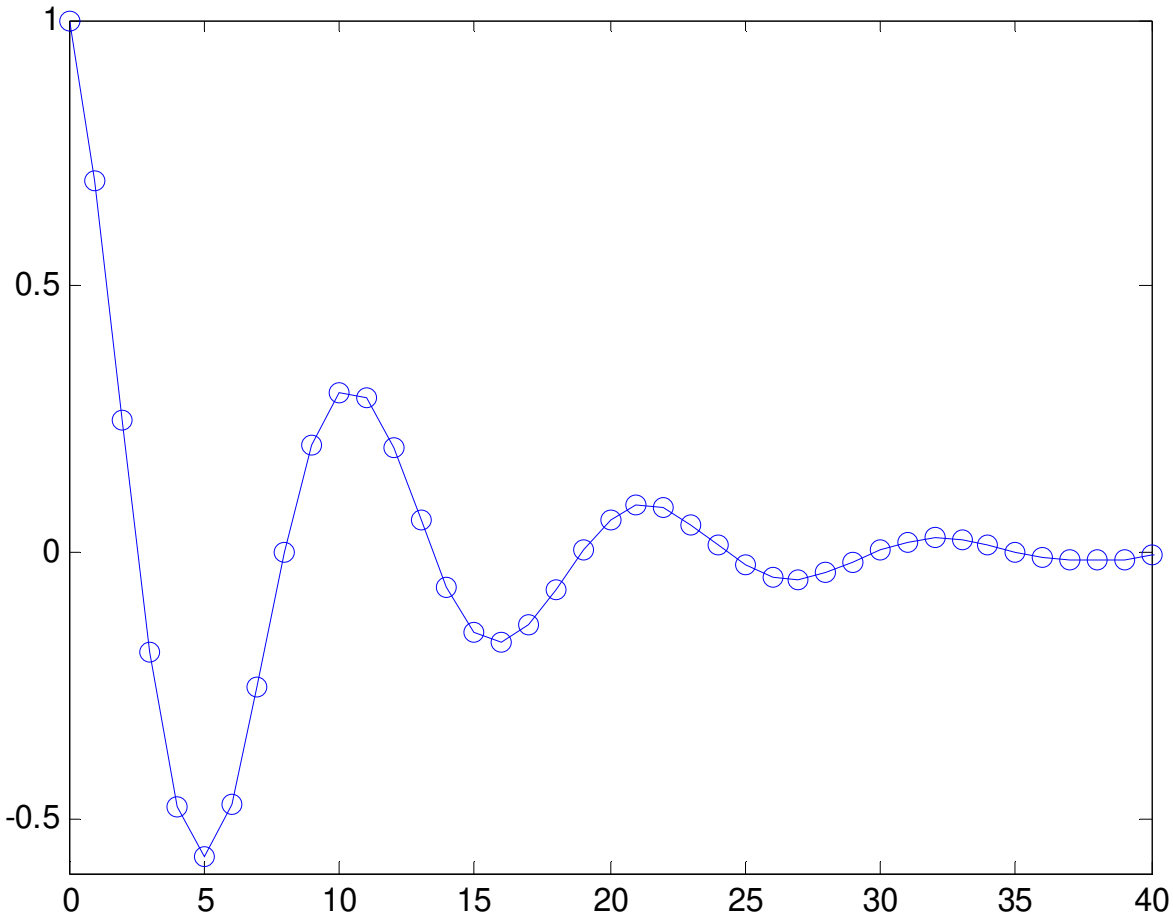
Respuesta a entrada cero

Respuesta a estado cero

Ejemplo

$$g[n]-1.5g[n-1]+0.8g[n-2] = f[n] \;\; g[-1] = 1,\; g[-2] = 1$$

Para f[n] = 0



Ejemplo

$$g[n] - 1.5g[n-1] + 0.8g[n-2] = f[n]$$

$$\begin{bmatrix} v_1[n+1] \\ v_2[n+1] \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.8 & 1.5 \end{bmatrix} \begin{bmatrix} v_1[n] \\ v_2[n] \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f[n]$$

$$g[n] = \begin{bmatrix} -0.8 & 1.5 \end{bmatrix} \begin{bmatrix} v_1[n] \\ v_2[n] \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -0.8 & 1.5 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad \mathbf{c} = \begin{bmatrix} -0.8 & 1.5 \end{bmatrix}$$

Ejemplo

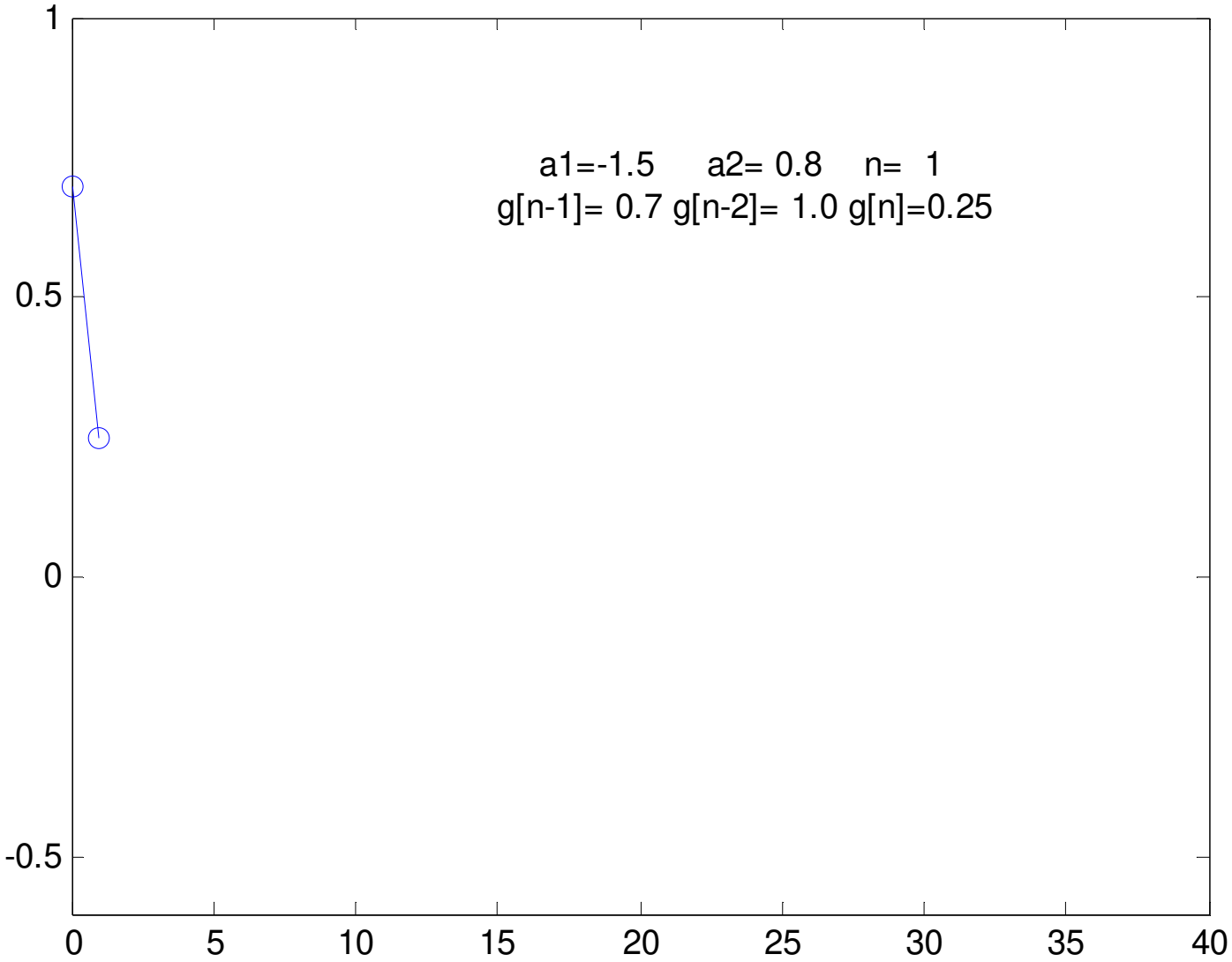
$$\mathbf{\Phi}[n] = \mathbf{A}^n$$

$$\mathbf{\Phi}[1] = \begin{bmatrix} 0 & 1 \\ -0.8 & 1.5 \end{bmatrix} \quad \mathbf{\Phi}[2] = \begin{bmatrix} -0.8 & 1.5 \\ -1.2 & 1.45 \end{bmatrix} \quad \mathbf{\Phi}[9] = \begin{bmatrix} 0.67 & -0.67 \\ 0.54 & -0.33 \end{bmatrix}$$

$$\mathbf{v}[1] = \mathbf{\Phi}[1]\mathbf{v}_o = \begin{bmatrix} 0 & 1 \\ -0.8 & 1.5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.7 \end{bmatrix}$$

$$g[1] = \mathbf{c}\mathbf{v}[1] = \begin{bmatrix} -0.8 & 1.5 \end{bmatrix} \begin{bmatrix} 1 \\ 0.7 \end{bmatrix} = 0.25$$

Ejemplo



Ejemplo

$$\mathbf{\Phi}[n] = \mathbf{A}^n$$

$$\mathbf{\Phi}[1] = \begin{bmatrix} 0 & 1 \\ -0.8 & 1.5 \end{bmatrix} \quad \mathbf{\Phi}[2] = \begin{bmatrix} -0.8 & 1.5 \\ -1.2 & 1.45 \end{bmatrix} \quad \mathbf{\Phi}[9] = \begin{bmatrix} 0.67 & -0.67 \\ 0.54 & -0.33 \end{bmatrix}$$

$$\mathbf{v}[9] = \mathbf{\Phi}[9]\mathbf{v}_o = \begin{bmatrix} 0.67 & -0.67 \\ 0.54 & -0.33 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}$$

$$g[9] = \mathbf{c}\mathbf{v}[9] = \begin{bmatrix} -0.8 & 1.5 \end{bmatrix} \begin{bmatrix} 0 \\ 0.2 \end{bmatrix} = 0.3$$

Ejemplo

