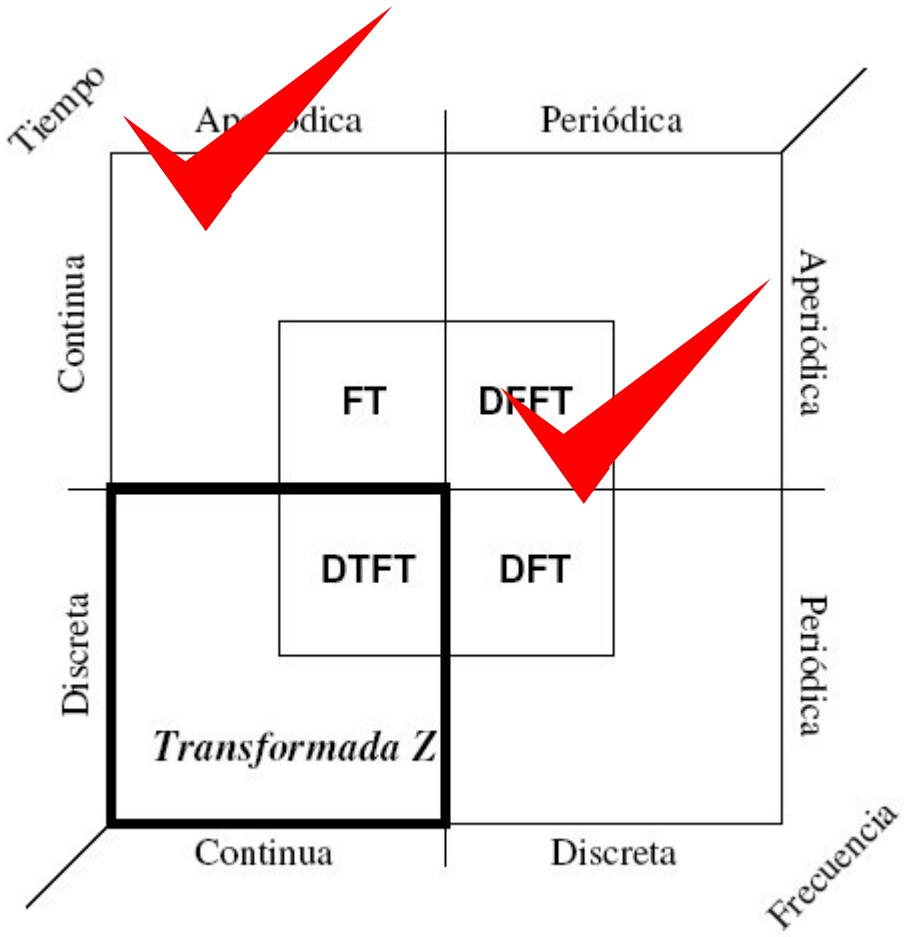
IEE 2103 Señales y Sistemas

Clase 7A: DTLT y Transformada Z

7. Transformada Z



DTLT ⟶ **ZT**

7.1 LT de tiempo discreto

Comenzando de la LT podemos definir la DTLT

Sea $Tf(nT) = f[n]$

con $F(s) = 0 \text{ para } |u| > 1/2T$

Sea

$$f_m(t) = \text{Ш}(t/T)f(t) = T \sum_{n=-\infty}^{\infty} f(nT)\delta(t - nT)$$

7.1 DTLT

La transformada de Laplace de $f_m(t)$ es

$$F_m(s) = T \sum_{n=-\infty}^{\infty} f(nT)\mathcal{L}\{\delta(t-nT)\}$$

$$= T \sum_{n=-\infty}^{\infty} f(nT)e^{-snT} \quad \alpha < \mathrm{Re}\{s\} < \beta$$

$$\tilde{F}(s) = \sum_{n=-\infty}^{\infty} f[n]e^{-snT} \quad \alpha < \mathrm{Re}\{s\} < \beta$$

7.1 DTLT inversa

La DTLT inversa se puede obtener de la LT inversa

$$f(t) = \frac{1}{i2\pi} \int_{c-i\infty}^{c+i\infty} F(s) e^{st}\, ds \quad \alpha < c < \beta$$

$$f(t) = \frac{1}{i2\pi} \int_{c}^{c+i2\pi/T} \tilde{F}(s) e^{st}\, ds \quad \alpha < c < \beta$$

$$f[n] = \frac{T}{i2\pi} \int_{c}^{c+i2\pi/T} \tilde{F}(s) e^{snT}\, ds \quad \alpha < c < \beta$$

7.2 DTLT y DTFT

La DTLT y la DTFT están relacionadas al igual que la LT con la FT

$$F(s) = F(\sigma + i2\pi u) = \mathcal{F}\{f(t)e^{-\sigma t}\} = \int_{-\infty}^{\infty} f(t)e^{-st}dt,$$

La DTFT es

$$\tilde{F}(u) = \sum_{n=-\infty}^{\infty} f[n]e^{-i2\pi unT}$$

$$\tilde{F}(s) = \tilde{F}(\sigma + i2\pi u) = \mathrm{DTFT}\{f[n]e^{-\sigma nT}\} = \sum_{n=-\infty}^{\infty} f[n]e^{-(\sigma + i2\pi u)nT}$$

$$\tilde{F}(s) = \sum_{n=-\infty}^{\infty} f[n]e^{-snT} \quad \alpha < \mathrm{Re}\{s\} < \beta$$

7.2 DTLT inversa y DTFT

La DTLT inversa se puede encontrar:

$$
\begin{aligned}
\mathrm{DTFT}\{f[n]e^{-\sigma nT}\} &= \tilde{F}(s) \\
f[n]e^{-\sigma nT} &= \mathrm{DTFT}^{-1}\{\tilde{F}(s)\} \\
f[n] &= e^{\sigma nT}\mathrm{DTFT}^{-1}\{\tilde{F}(s)\} \\
&= e^{\sigma nT}T\int_0^{1/T}\tilde{F}(\sigma+i2\pi u)e^{i2\pi unT}\,du \\
&= T\int_0^{1/T}\tilde{F}(\sigma+i2\pi u)e^{(\sigma+i2\pi u)nT}\,du
\end{aligned}
$$

$$
f[n] = \frac{T}{i2\pi}\int_c^{c+i2\pi/T}\tilde{F}(s)e^{snT}\,ds \quad \alpha < c < \beta
$$

7.3 Transformada Z

DTLT: $$\tilde{F}(s) = \sum_{n=-\infty}^{\infty} f[n] e^{-snT}$$

Sustituyendo $$z = e^{sT}$$

ZT: $$F(z) = \sum_{n=-\infty}^{\infty} f[n] z^{-n}$$

Falta definir la región de convergencia

7.3 Transformada Z

$$F(z) = \sum_{n=-\infty}^{\infty} f[n] z^{-n}.$$

$$\delta[n-1] \longrightarrow e^{-sT} \qquad \text{(DTLT)}$$

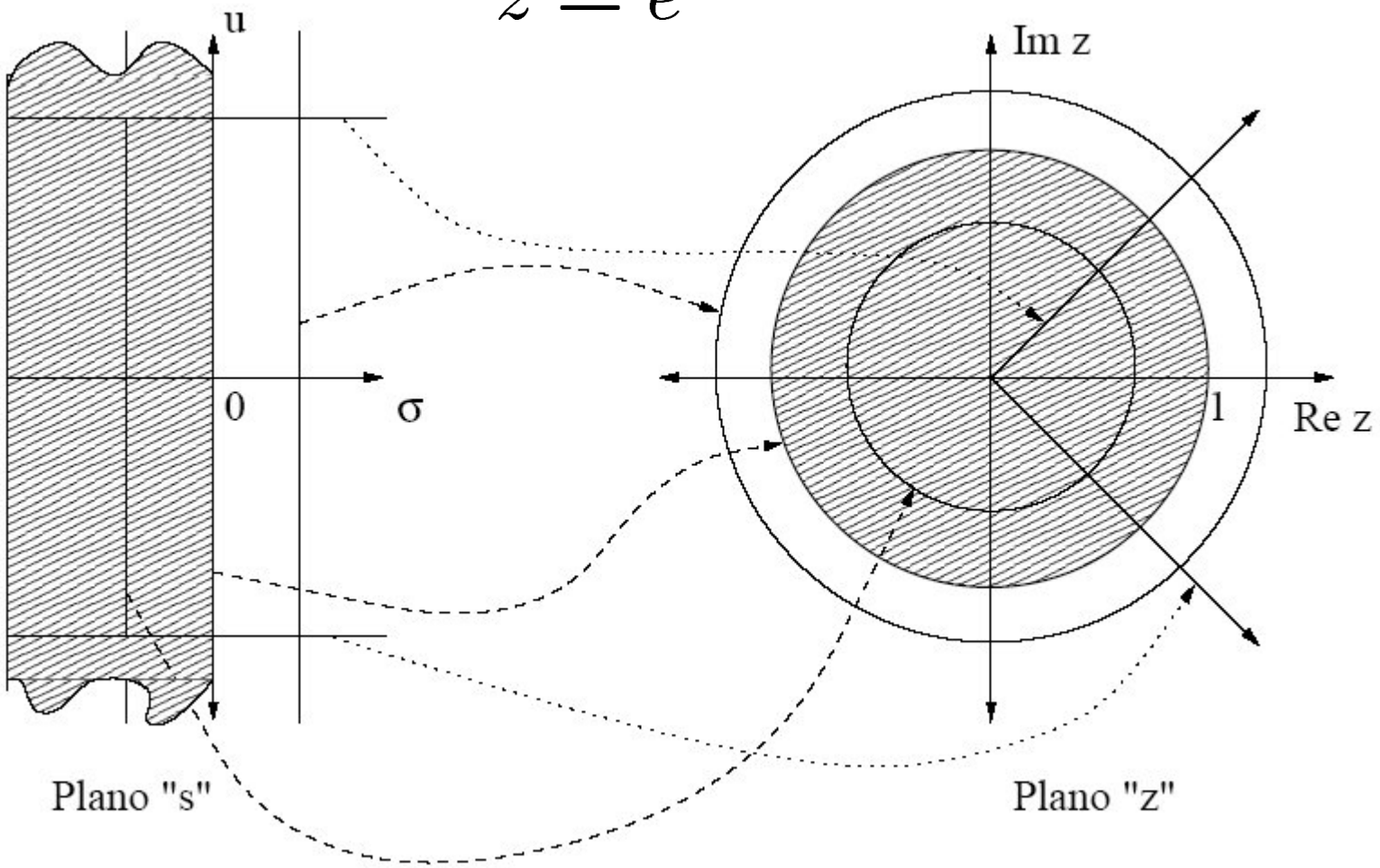
implica que z^{-1} está asociado a un retardo de uno

$$f[n] \longrightarrow F(z)$$

$$(3, 4, \underline{5}, 1, 3) \longrightarrow 3z^2 + 4z + 5 + z^{-1} + 3z^{-2}$$

7.3 Transformada Z

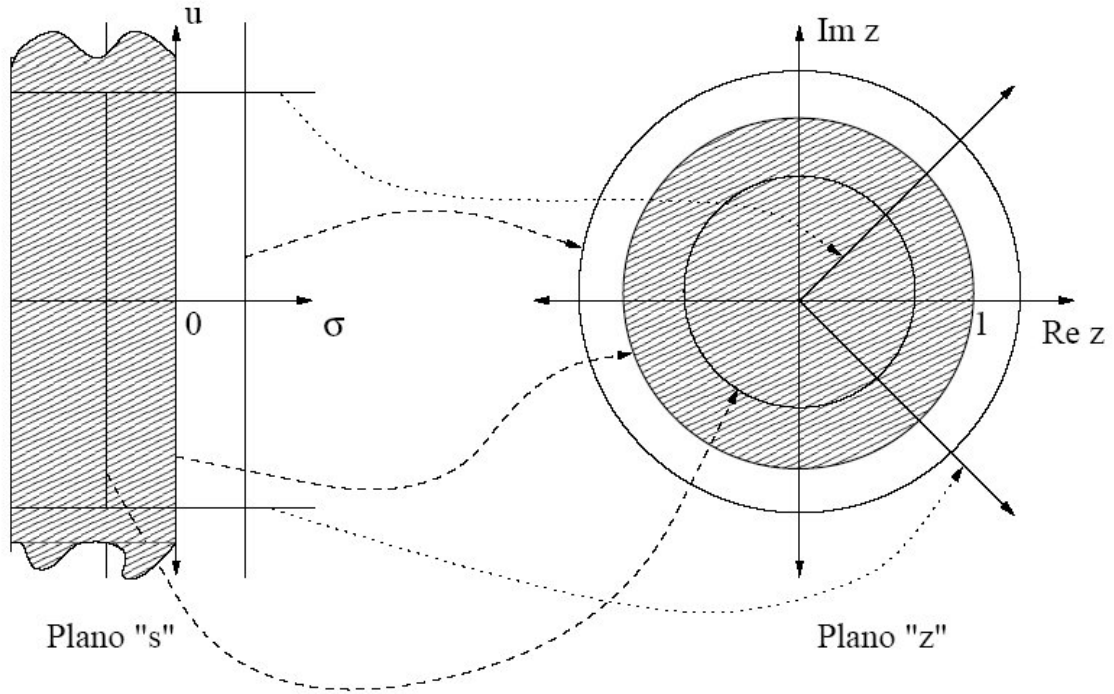
$$z = e^{sT}$$



7.3 Transformada Z

Analicemos las implicancias de

$$z = e^{sT}$$



$$\tilde{F}(s) = \sum_{n=-\infty}^{\infty} f[n]e^{-snT}$$

$$F(z) = \sum_{n=-\infty}^{\infty} f[n]z^{-n}$$

$$\tilde{F}(u) = \sum_{n=-\infty}^{\infty} f[n]e^{-i2\pi unT}$$

7.3 Transformada Z

La ZT definida de la DTFT

$$z = e^{sT} = e^{\sigma T} e^{i2\pi uT}$$

$$F(z) = \sum_{n=-\infty}^{\infty} f[n] z^{-n}$$

$$F(re^{i2\pi uT}) = \sum_{n=-\infty}^{\infty} f[n] r^{-n} e^{-i2\pi unT}$$

$$F(re^{i2\pi uT}) = \mathsf{DTFT}\{f[n] r^{-n}\}$$

7.3 Transformada Z

$$f[n] \longrightarrow F(z)$$

Si la región de convergencia incluye al círculo unitario, entonces

$$\tilde{F}(u) = F(e^{i2\pi uT})$$

La DTFT es la ZT evaluada en el círculo unitario.