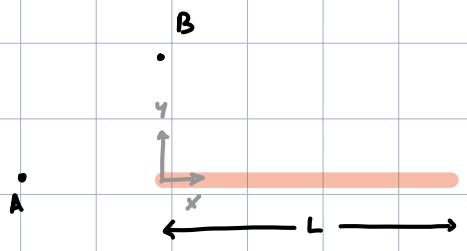
Pauta P1 I1 2023/01



(a) (1pto) Calcular $\vec{E}(A)$

$\lambda(x) = \beta x \quad dq(x') = \lambda(x')\,dx' = \beta x'\,dx'$ ← 0.3 pt

$$\vec{E}(A) = \vec{E}(\vec{r} = -a\hat{x}) = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{\beta x'\,dx'\,(-a\hat{x} - x'\hat{x})}{|-a\hat{x} - x'\hat{x}|^3} = \frac{\beta}{4\pi\varepsilon_0}\int_0^L dx' \frac{-x'(a+x')\hat{x}}{(a+x')^3} = \frac{\beta\hat{x}}{4\pi\varepsilon_0}\int_0^L dx' \frac{-x'}{(a+x')^2}$$

0.3 pt (integrando) — 0.3 pt (resultado)

$u = a + x' \quad du = dx'$

$$= -\frac{\beta\hat{x}}{4\pi\varepsilon_0}\int_a^{a+L} du\,\frac{u-a}{u^2} = -\frac{\beta\hat{x}}{4\pi\varepsilon_0}\left\{\int_a^{a+L}\frac{du}{u} - \int_a^{a+L} du\,\frac{a}{u^2}\right\} = -\frac{\beta\hat{x}}{4\pi\varepsilon_0}\left\{\ln\left(\frac{a+L}{a}\right) - \left(-\frac{a}{a+L} + \frac{a}{a}\right)\right\}$$

OJO! update 30/5

$$= -\frac{\beta\hat{x}}{4\pi\varepsilon_0}\left\{\ln\left(\frac{a+L}{a}\right) - 1 + \frac{a}{a+L}\right\}$$

0.1 pt

(b) (1 pto) Calcular $V(A)$

asumimos ref $r \to \infty$ 0,1 pt

$u = a + x' \quad du = dx'$

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{\beta x'\,dx'}{|-a\hat{x} - x'\hat{x}|} = \frac{\beta}{4\pi\varepsilon_0}\int_0^L \frac{x'}{(a+x')}dx' = \frac{\beta}{4\pi\varepsilon_0}\left\{\int_a^{a+L}\frac{u-a}{u}\,du\right\} = \frac{\beta}{4\pi\varepsilon_0}\left\{\int_a^{a+L} 1 - \frac{a}{u}\,du\right\}$$

0,4 pt — 0,4 pt

$$= \frac{\beta}{4\pi\varepsilon_0}\left\{a + L - a - a\ln\left(\frac{L+a}{a}\right)\right\} = \frac{\beta}{4\pi\varepsilon_0}\left\{L - a\ln\left(\frac{L+a}{a}\right)\right\}$$

0.1 pt

(c) Calcular β tq $V(B) - V(A) = \dfrac{Q_0}{4\pi\varepsilon_0 L}$

necesitamos $V(B)$

$u = b^2 + x'^2 \quad du = 2x'dx'$

$$V(B) = V(b\hat{y}) = \frac{1}{4\pi\varepsilon_0}\int_0^L \frac{\beta x'\,dx'}{|b\hat{y} - x'\hat{x}|} = \frac{\beta}{4\pi\varepsilon_0}\int_0^L \frac{x'\,dx'}{(b^2+x'^2)^{1/2}} = \frac{\beta}{4\pi\varepsilon_0}\frac{1}{2}\int_{b^2}^{b^2+L^2}\frac{du}{u^{1/2}}$$

0.4 pt — 0.4 pt — $u^{-1/2}$ — $(u^{1/2})' = \frac{1}{2}u^{1/2-1} = \frac{1}{2}\frac{1}{u^{1/2}}$

$$= \frac{\beta}{4\pi\varepsilon_0}\frac{1}{2}\,2u^{1/2}\Big]_{b^2}^{b^2+L^2} = \frac{\beta}{4\pi\varepsilon_0}\left\{\sqrt{b^2+L^2} - |b|\right\}$$

0.1 pt

$$\left|V(B) - V(A)\right| = \frac{\beta}{4\pi\varepsilon_0}\left(L - a\ln\left(\frac{L+a}{a}\right) - \sqrt{b^2+L^2} + |b|\right) = \frac{Q_0}{4\pi\varepsilon_0 L}$$

acá puede ser $V(A) - V(B)$ también

$$\beta = \frac{Q_0}{L}\left(L - a\ln\left(\frac{L+a}{a}\right) - \sqrt{b^2+L^2} + b\right)^{-1}$$

0.1 pt