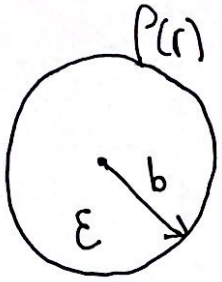
Ayudantía 12

1.



$$P(r)\begin{cases} P_0 r & 0 \le r \le b \\ 0 & r > b \end{cases}$$

a) ρ_p y ∇_p?

- Para $r>b$ es vacío, no existe vector de polarización.
- Para $r \le b$:

$$\oint D \cdot ds = Q_e = \int_V P_0 r \, dv \xrightarrow{r^2 \sin\theta \, dr\, d\theta\, d\phi}$$

$$D(r) \cdot 4\pi r^2 = \frac{4\pi r^2 \pi P_0}{4}$$

$$\vec{D}(r) = \frac{P_0 r^2}{4}\hat{r} \rightarrow \boxed{\vec{E}(r) = \frac{P_0 r^2}{4\varepsilon}\hat{r}}$$

Luego: $\vec{P} = \vec{D} - \varepsilon_0 \vec{E}$

$$\vec{P} = \frac{(\varepsilon - \varepsilon_0) P_0 r^2}{4\varepsilon}\hat{r}$$

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Luego, la densidad de carga superficial de polarización:

$$\sigma_p = \vec{P} \cdot \hat{n}\Big|_{r=b}$$

$$\boxed{\sigma_p = \frac{(\varepsilon - \varepsilon_0) P_0 b^2}{4\varepsilon}}$$

La densidad de carga volumétrica es:

$$\rho_p = -\nabla \cdot \vec{P}$$

$$\rho_p = -\frac{1}{r^2}\frac{\partial}{\partial r}(P_r \cdot r^2)$$

Recordar:

$$\nabla \cdot \vec{F} = \frac{1}{r^2}\frac{\partial (r^2 F_r)}{\partial r}$$

$$\rho_p = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(\frac{(\varepsilon - \varepsilon_0) P_0 r^4}{4\varepsilon}\right) = -\frac{1}{r^2}\frac{(\varepsilon - \varepsilon_0) P_0 r^3}{\varepsilon}$$

$$\boxed{\rho_p = \frac{(\varepsilon_0 - \varepsilon) P_0 r}{\varepsilon}}$$

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2. $U = -p \cdot \vec{E}$

Con $E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$, reemplazando:

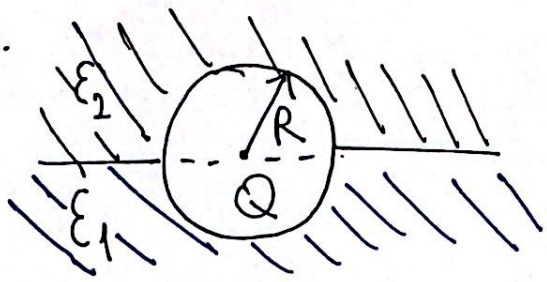
$$\boxed{U = -\frac{Q}{4\pi\varepsilon_0 r^2}\cdot \vec{p}\,\hat{r}}$$

El torque es:

$$\boxed{\vec{T} = p \times E(r) = \frac{Q}{4\pi\varepsilon_0 r^2}\,\vec{p} \times \hat{r}}$$

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3.



a) ¿E? en todo el espacio:

- $r < R$: $\vec{E} = 0$ → Conductor.
- $r > R$:

$$\underbrace{\oint D \cdot ds}_{} = Q_\ell \qquad \text{Con } E_1 = E_2 = E$$

$$\underbrace{D_1(r)} \cdot 2\pi r^2 + \underbrace{D_2(r)} \cdot 2\pi r^2 = Q$$

$\varepsilon_1 E(r)$ $\qquad$ $\varepsilon_2 E(r)$

Luego : $\varepsilon_1 E \cdot 2\pi r^2 + \varepsilon_2 E \, 2\pi r^2 = Q$

$$\boxed{E(r) = \frac{Q}{2\pi r^2(\varepsilon_1 + \varepsilon_2)}}$$

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b) $\sigma_{l1} = \vec{D_1} \cdot \hat{n} = \dfrac{\varepsilon_1 Q}{2\pi R^2(\varepsilon_1+\varepsilon_2)} \hat{r} \cdot \hat{r}$

$\sigma_{l2} = \vec{D_2} \cdot \hat{n} = \dfrac{\varepsilon_2 Q}{2\pi R^2(\varepsilon_1+\varepsilon_2)} \hat{r} \cdot \hat{r}$

La de polarización:

$$\vec{D} = \varepsilon_0 E + P \rightarrow \vec{P} = (\varepsilon_1 - \varepsilon_0)\vec{E}$$

$$P_1 = (\varepsilon_1 - \varepsilon_0)\vec{E} = \frac{(\varepsilon_1 - \varepsilon_0) \cdot Q}{2\pi r^2(\varepsilon_1+\varepsilon_2)} \hat{r}$$

$\sigma_{p1} = P_1 \cdot \hat{n}$

$\sigma_{p2} = P_2 \cdot \hat{n}$ //

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