Guia 3

Andrés Yenes S.

1.

4-120 A cylinder is initially filled with helium gas at a specified state. Helium is compressed polytropically to a specified temperature and pressure. The heat transfer during the process is to be determined.

Assumptions **1** Helium is an ideal gas with constant specific heats. **2** The cylinder is stationary and thus the kinetic and potential energy changes are negligible. **3** The thermal energy stored in the cylinder itself is negligible. **4** The compression or expansion process is quasi-equilibrium.

Properties The gas constant of helium is $R = 2.0769$ kPa.m^3/kg.K (Table A-1). Also, $c_v = 3.1156$ kJ/kg.K (Table A-2).

Analysis The mass of helium and the exponent n are determined to be

$$m = \frac{P_1 V_1}{RT_1} = \frac{(150\ \text{kPa})(0.5\ \text{m}^3)}{(2.0769\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(293\ \text{K})} = 0.123\ \text{kg}$$

$$\frac{P_1 V_1}{RT_1} = \frac{P_2 V_2}{RT_2} \longrightarrow V_2 = \frac{T_2 P_1}{T_1 P_2} V_1 = \frac{413\ \text{K}}{293\ \text{K}} \times \frac{150\ \text{kPa}}{400\ \text{kPa}} \times 0.5\ \text{m}^3 = 0.264\ \text{m}^3$$

$$P_2 V_2^n = P_1 V_1^n \longrightarrow \left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^n \longrightarrow \frac{400}{150} = \left(\frac{0.5}{0.264}\right)^n \longrightarrow n = 1.536$$

Then the boundary work for this polytropic process can be determined from

$$W_{b,in} = -\int_1^2 P\,dV = -\frac{P_2 V_2 - P_1 V_1}{1-n} = -\frac{mR(T_2 - T_1)}{1-n}$$

$$= -\frac{(0.123\ \text{kg})(2.0769\ \text{kJ/kg}\cdot\text{K})(413-293)\text{K}}{1-1.536} = 57.2\ \text{kJ}$$

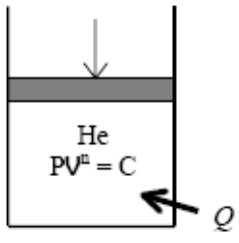
We take the contents of the cylinder as the system. This is a closed system since no mass enters or leaves. Taking the direction of heat transfer to be to the cylinder, the energy balance for this stationary closed system can be expressed as

$$\underbrace{E_{in} - E_{out}}_{\text{Net energy transfer by heat, work, and mass}} = \underbrace{\Delta E_{system}}_{\text{Change in internal, kinetic, potential, etc. energies}}$$

$$Q_{in} + W_{b,in} = \Delta U = m(u_2 - u_1)$$

$$Q_{in} = m(u_2 - u_1) - W_{b,in}$$

$$= mc_v(T_2 - T_1) - W_{b,in}$$



Substituting,

$$Q_{in} = (0.123\ \text{kg})(3.1156\ \text{kJ/kg}\cdot\text{K})(413 - 293)\text{K} - (57.2\ \text{kJ}) = \mathbf{-11.2\ kJ}$$

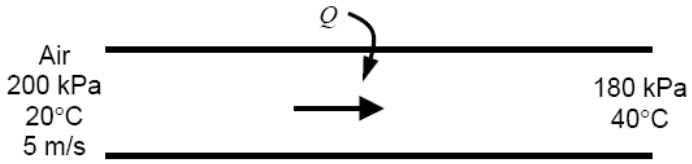
The negative sign indicates that heat is lost from the system.

2.

-

3.

5-15 Air flows through a pipe. Heat is supplied to the air. The volume flow rates of air at the inlet and exit, the velocity at the exit, and the mass flow rate are to be determined.



Properties The gas constant for air is 0.287 kJ/kg.K (Table A-2).

Analysis (*a*) (*b*) The volume flow rate at the inlet and the mass flow rate are

$$\dot{V}_1 = A_c V_1 = \frac{\pi D^2}{4} V_1 = \frac{\pi (0.28\ \text{m})^2}{4} (5\ \text{m/s}) = \mathbf{0.3079\ m^3/s}$$

$$\dot{m} = \rho_1 A_c V_1 = \frac{P_1}{RT_1} \frac{\pi D^2}{4} V_1 = \frac{(200\ \text{kPa})}{(0.287\ \text{kJ/kg.K})(20+273\ \text{K})} \frac{\pi (0.28\ \text{m})^2}{4} (5\ \text{m/s}) = \mathbf{0.7318\ kg/s}$$

(c) Noting that mass flow rate is constant, the volume flow rate and the velocity at the exit of the pipe are determined from

$$\dot{V}_2 = \frac{\dot{m}}{\rho_2} = \frac{\dot{m}}{\frac{P_2}{RT_2}} = \frac{0.7318\ \text{kg/s}}{\frac{(180\ \text{kPa})}{(0.287\ \text{kJ/kg.K})(40+273\ \text{K})}} = \mathbf{0.3654\ m^3/s}$$

$$V_2 = \frac{\dot{V}_2}{A_c} = \frac{0.3654\ \text{m}^3/\text{s}}{\frac{\pi (0.28\ \text{m})^2}{4}} = \mathbf{5.94\ m/s}$$

4.

3-67 Heat is supplied to a rigid tank that contains water at a specified state. The volume of the tank, the final temperature and pressure, and the internal energy change of water are to be determined.

Properties The saturated liquid properties of water at 200°C are: $v_f = 0.001157\ \text{m}^3/\text{kg}$ and $u_f = 850.46$ kJ/kg (Table A-4).

Analysis (*a*) The tank initially contains saturated liquid water and air. The volume occupied by water is

$$V_1 = mv_1 = (1.4\ \text{kg})(0.001157\ \text{m}^3/\text{kg}) = 0.001619\ \text{m}^3$$

which is the 25 percent of total volume. Then, the total volume is determined from

$$V = \frac{1}{0.25}(0.001619) = \mathbf{0.006476\ m^3}$$

(*b*) Properties after the heat addition process are

$$v_2 = \frac{V}{m} = \frac{0.006476\ \text{m}^3}{1.4\ \text{kg}} = 0.004626\ \text{m}^3/\text{kg}$$

$$\left.\begin{array}{l} v_2 = 0.004626\ \text{m}^3/\text{kg} \\ x_2 = 1 \end{array}\right\} \begin{array}{l} T_2 = \mathbf{371.3°C} \\ P_2 = \mathbf{21{,}367\ kPa} \\ u_2 = 2201.5\ \text{kJ/kg} \end{array} \quad \text{(Table A-4 or A-5 or EES)}$$

(*c*) The total internal energy change is determined from

$$\Delta U = m(u_2 - u_1) = (1.4\ \text{kg})(2201.5 - 850.46)\ \text{kJ/kg} = \mathbf{1892\ kJ}$$

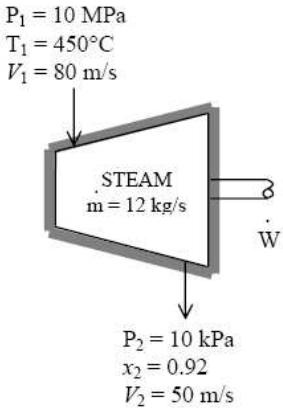
5.

5-49 Steam expands in a turbine. The change in kinetic energy, the power output, and the turbine inlet area are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time. **2** Potential energy changes are negligible. **3** The device is adiabatic and thus heat transfer is negligible.

Properties From the steam tables (Tables A-4 through 6)

$$\left.\begin{matrix} P_1 = 10\ \text{MPa} \\ T_1 = 450°\text{C} \end{matrix}\right\} \begin{matrix} v_1 = 0.029782\ \text{m}^3/\text{kg} \\ h_1 = 3242.4\ \text{kJ/kg} \end{matrix}$$



and

$$\left.\begin{matrix} P_2 = 10\ \text{kPa} \\ x_2 = 0.92 \end{matrix}\right\} h_2 = h_f + x_2 h_{fg} = 191.81 + 0.92 \times 2392.1 = 2392.5\ \text{kJ/kg}$$

Analysis (a) The change in kinetic energy is determined from

$$\Delta ke = \frac{V_2^2 - V_1^2}{2} = \frac{(50\ \text{m/s})^2 - (80\ \text{m/s})^2}{2}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = -1.95\ \text{kJ/kg}$$

(*b*) There is only one inlet and one exit, and thus $\dot{m}_1 = \dot{m}_2 = \dot{m}$. We take the turbine as the system, which is a control volume since mass crosses the boundary. The energy balance for this steady-flow system can be expressed in the rate form as

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0\ \text{(steady)}}}_{\substack{\text{Rate of change in internal, kinetic,} \\ \text{potential, etc. energies}}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}(h_1 + V_1^2/2) = \dot{W}_{\text{out}} + \dot{m}(h_2 + V_2^2/2) \quad (\text{since } \dot{Q} \cong \Delta pe \cong 0)$$

$$\dot{W}_{\text{out}} = -\dot{m}\left(h_2 - h_1 + \frac{V_2^2 - V_1^2}{2}\right)$$

Then the power output of the turbine is determined by substitution to be

$$\dot{W}_{\text{out}} = -(12\ \text{kg/s})(2392.5 - 3242.4 - 1.95)\text{kJ/kg} = \mathbf{10.2\ MW}$$

(*c*) The inlet area of the turbine is determined from the mass flow rate relation,

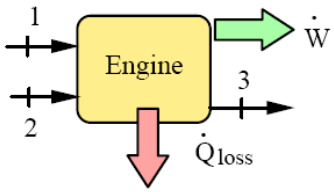
$$\dot{m} = \frac{1}{v_1} A_1 V_1 \longrightarrow A_1 = \frac{\dot{m} v_1}{V_1} = \frac{(12\ \text{kg/s})(0.029782\ \text{m}^3/\text{kg})}{80\ \text{m/s}} = \mathbf{0.00447\ m^2}$$

6.

Two steady flows of air enters a control volume, shown in Fig. P6.80. One is 0.025 kg/s flow at 350 kPa, 150°C, state 1, and the other enters at 450 kPa, 15°C, both flows with low velocity. A single flow of air exits at 100 kPa, −40°C, state 3. The control volume rejects 1 kW heat to the surroundings and produces 4 kW of power. Neglect kinetic energies and determine the mass flow rate at state 2.

Solution:

C.V. Steady device with two inlet and one exit flows, we neglect kinetic energies. Notice here the Q is rejected so it goes out.



Continuity Eq.6.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3 = 0.025 + \dot{m}_2$

Energy Eq.6.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{W}_{CV} + \dot{Q}_{loss}$

Substitute the work and heat transfer into the energy equation and use constant heat capacity

$$0.025 \times 1.004 \times 423.2 + \dot{m}_2 \times 1.004 \times 288.2$$

$$= (0.025 + \dot{m}_2)\ 1.004 \times 233.2\ + 4.0 + 1.0$$

Now solve for $\dot{m}_2$.

$$\dot{m}_2 = \frac{4.0 + 1.0 + 0.025 \times 1.004 \times (233.2 - 423.2)}{1.004\ (288.2 - 233.2)}$$

Solving, $\dot{m}_2 = \mathbf{0.0042\ kg/s}$

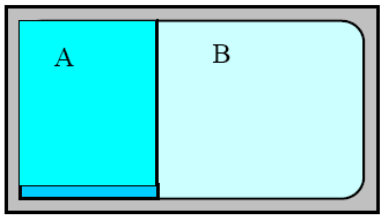
7. –

8.

A rigid tank is divided into two rooms by a membrane, both containing water, shown in Fig. P5.58. Room A is at 200 kPa, $v = 0.5$ m^3/kg, $V_A = 1$ m^3, and room B contains 3.5 kg at 0.5 MPa, 400°C. The membrane now ruptures and heat transfer takes place so the water comes to a uniform state at 100°C. Find the heat transfer during the process.

Solution:

C.V.: Both rooms A and B in tank.



Continuity Eq.: $m_2 = m_{A1} + m_{B1}$;

Energy Eq.: $m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = {}_1Q_2 - {}_1W_2$

State 1A: (P, v) Table B.1.2, $m_{A1} = V_A/v_{A1} = 1/0.5 = 2$ kg

$$x_{A1} = \frac{v - v_f}{v_{fg}} = \frac{0.5 - 0.001061}{0.88467} = 0.564$$

$$u_{A1} = u_f + x\, u_{fg} = 504.47 + 0.564 \times 2025.02 = 1646.6 \text{ kJ/kg}$$

State 1B: Table B.1.3, $v_{B1} = 0.6173$, $u_{B1} = 2963.2$, $V_B = m_{B1}v_{B1} = 2.16$ m^3

Process constant total volume: $V_{tot} = V_A + V_B = 3.16$ m^3 and ${}_1W_2 = \emptyset$

$$m_2 = m_{A1} + m_{B1} = 5.5 \text{ kg} \quad \Rightarrow \quad v_2 = V_{tot}/m_2 = 0.5746 \text{ m}^3\text{/kg}$$

State 2: T_2 , v_2 $\Rightarrow$ Table B.1.1 two-phase as $v_2 < v_g$

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.5746 - 0.001044}{1.67185} = 0.343 ,$$

$$u_2 = u_f + x\, u_{fg} = 418.91 + 0.343 \times 2087.58 = 1134.95 \text{ kJ/kg}$$

Heat transfer is from the energy equation

$${}_1Q_2 = m_2u_2 - m_{A1}u_{A1} - m_{B1}u_{B1} = \mathbf{-7421\ kJ}$$

9.

-

10.

A diffuser, shown in Fig. P6.36, has air entering at 100 kPa, 300 K, with a velocity of 200 m/s. The inlet cross-sectional area of the diffuser is 100 mm^2. At the exit, the area is 860 mm^2, and the exit velocity is 20 m/s. Determine the exit pressure and temperature of the air.

Solution:

Continuity Eq.6.3: $\dot{m}_i = A_i\mathbf{V}_i/v_i = \dot{m}_e = A_e\mathbf{V}_e/v_e$,

Energy Eq.(per unit mass flow)6.13: $h_i + \frac{1}{2}\mathbf{V}_i^2 = h_e + \frac{1}{2}\mathbf{V}_e^2$

$h_e - h_i = \frac{1}{2} \times 200^2/1000 - \frac{1}{2} \times 20^2/1000 = 19.8$ kJ/kg

$T_e = T_i + (h_e - h_i)/C_p = 300 + 19.8/1.004 =$ **319.72 K**

Now use the continuity equation and the ideal gas law

$$v_e = v_i\left(\frac{A_e\mathbf{V}_e}{A_i\mathbf{V}_i}\right) = (RT_i/P_i)\left(\frac{A_e\mathbf{V}_e}{A_i\mathbf{V}_i}\right) = RT_e/P_e$$

$$P_e = P_i\left(\frac{T_e}{T_i}\right)\left(\frac{A_i\mathbf{V}_i}{A_e\mathbf{V}_e}\right) = 100\left(\frac{319.72}{300}\right)\left(\frac{100 \times 200}{860 \times 20}\right) = \mathbf{123.92\ kPa}$$

